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HOW THE LOGIC AND PRAGMATICS

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Abstract

This paper sets forth that sinking funds foster corporate governance, either when they intend to build up the principal of bonds and financial hybrids to be repaid at maturity date, or to plan ahead the purchase of fixed assets in the future. To lay foundations, firstly we expand on the logic of sinking funds, by reviewing the standard model of capital formation. Proven drawbacks of this model, however, pave the way for our proposal of undertaking a portfolio management approach for which we furnish an iterative resetting program that deals with unavoidable imbalances of the underlying portfolio. Secondly, we develop the pragmatics of sinking funds, which focus on the choice problem attached to sinking funds and the fiduciary role expected from an appointed portfolio manager. Lastly, we move on to a protocol with suitable covenants to be embedded in a bond placement, so as to enhance the governance of those organizations that dare to avail themselves of sinking funds.

JEL : G38, G30, G32

Key words: sinking fund, corporate governance, bonds placement, financial hybrids, fixed assets, capital formation, portfolio management.

INTRODUCTION

For almost a century, scholars have given scant regard to sinking funds. If we seek for references in JSTOR, for instance, we will find that they were surprisingly sparse in the last hundred years, the fewest related to fixed assets provisions, the remaining addressing either bonds or preferred stock redemptions.

Among the former lot, we come upon an extremely noticeable essay written by Alfred Chandler, way back in 1913, which delved into diverse amortization mechanisms. Representatives of the latter lot are two contributions that match the subject matter of our present research. Firstly, an old but still readable review of British and American policies of funding and retrieving government debt through sinking funds along the XVIII and XIX centuries (Edward Ross, 1892). Secondly, an article by Lawrence Wilsey (1947), dealing with the use of sinking funds in Preferred Stock issues in the United States[1].

Up to our knowledge, however, this is the first paper that extensively underscores the close link between sinking funds and corporate governance. The roadmap will be the following:

In section 1, we lay bare the logic of sinking funds, by expanding on the underlying mathematics of the standard model for capital formation processes and stressing some of its downsides. Next, we argue that a more sensible approach to sinking funds naturally stems from portfolio building. Lastly, we develop a very simple algorithm that deals with the resetting of the portfolio's valuation.

It is for section 2 to put forward the pragmatics of a sinking fund grounded on two distinctive issues. On the one hand, the choice problem of a financial vehicle that could turn out a targeted capital stock in the future. On the other, we discuss the role of a fiduciary agent in managing the sinking fund.

In section 3, we are going to show how sinking funds play a distinctive part on the governance and practices of any organization in the private sector. Afterwards, it will be set down a protocol for sinking funds in the case of bonds' placements.

[1] We may also refer here to our own contribution (Apreda, 2004) devoted to one- and two-tiered convertible preferred stock.

Last of all, three appendices furnish with mathematical foundations to sections 1 and 2.

1. THE LOGIC OF SINKING FUNDS

What are the inner structure, overall purposes, and the computational method that sinking funds bid for? In other words, what is the logic behind these financial vehicles? To start working with these topics, we need to bring forward a definition functional to this paper.

Definition 1 ***Sinking Funds***

*By **Sinking Fund** we understand a process of capital formation in the future, adding up to the amount*

$$FV\ (0;\ N)$$

that will evolve through a given planning horizon,

$$H = [0;\ N]$$

*divided in **N** periods of equal size*

$$[k;\ k + 1] \quad ; \quad k = 0, 1, 2, \ldots\ldots, N - 1$$

*by means of the iterative allocation, at the beginning of each period along **H**, of a well defined pattern of cash flows*

$$CF\ (\ k\ ) \quad ; \quad k = 0, 1, 2, \ldots., N - 1$$

which will earn expected returns

$$R(k;\ k + 1)$$

*period after period, till maturity at date **N**.*

To place this definition into perspective, some qualifications must be borne in mind.

> Firstly, there is no denying the fact that alternative statements of meaning are available: for instance, periods may have unequal length, or cash flows may be disbursed at the end of each period instead of the beginning.

Secondly, albeit the mainstream notion of sinking funds regards them as financial tools to retire debt year after year[2], a different view will be stressed in this paper. Actually, we are going to focus on sinking funds pertaining capital formation processes not only linked with debt, but also with financial hybrids[3] or capital assets. Furthermore, and in all cases, the kind of sinking fund we are interested in will evolve till maturity date without any sort of progressive redemption.

Lastly, as we will develop in sections 2 and 3, our definition will allow for an in-built feature consisting in the fact that the sinking fund is managed by an externally fiduciary agent, and not from within the company.

In current usage, two criteria have prevailed when assessing which cash flows would be the most operational for the funding of a sinking fund[4]:

- either they are framed upon the company's earnings during the preceding year, after accounting for taxes, preferred and ordinary dividends;

- or they are measured up after taxes, but before preferred and ordinary dividends, which liken them to a fixed charge, the sort of contractual payments we usually find in debt contracts.

Although not denying the advantages of the first criterion, we rather advocate for the second one, since it prioritizes a tighter budget constraint and, moreover, better governance.

It must be noticed that the sinking fund of Definition 1 stands apart from the prevailing meaning of a **contingency fund**. The latter consists in an amount of money (or string of cash flows, eventually) that a company sets aside to pay for a possible expense or loss in the future. The contingency attribute points to the fact that money outlays

[2] By means of contractual patterns of anticipated redemption mechanisms, from lottery devices to call-provision constraints embedded as covenants in bank loans and bond contracts.

[3] A corporate bond, or preferred stock with an attached warrant, for instance. In the case of convertible bonds or preferred stock, the implicit option could impair the sinking fund and increase the cost of the vehicle. More on this in footnote 5.

[4] In his insightful paper on amortization Chandler (1913) stressed that, in order to extinguish debt through sinking funds, it should be used the revenue above the expenses.

would be solely triggered off on the grounds of a well-defined event expected in the future. In contradistinction, the sinking fund can be seen as a purposeful design of cash flows to meet a forward commitment whose monetary value does not depend on random states of the world.

To round off this section, a question might be arisen as to whether we could set up a contingency fund by means of a sinking fund. We can do it, by all means, because the former is a future capital that we would need if the defining contingency took place, while the latter is a suitable mechanism by which the required capital accrues as time passes by. On the other hand, if the contingency did not happen, then the sinking fund would be employed in alternative allocations at maturity date[5].

1.1 THE UNDERLYING MATHEMATICS

How do we establish a future amount of money by setting aside cash flows that will earn returns over certain span of time? The simplest model of sinking fund that has been used for centuries springs from the following assumptions:

- cash flows are constant, that is to say

$$\mathbf{CF (k) = C \quad ; \quad k = 0, 1, 2, \ldots\ldots , N - 1}$$

- period after period, cash flows earn a fixed rate of return that amounts to a suitable rate of interest in the money market:

$$\mathbf{R(k; k + 1) = I}$$

With such constraints, it can easily be proved[6] that the future capital will add up to

$$\mathbf{FV(0; N) = [C / I] . [(1 + I)^N - 1] . [1 + I]} \tag{1}$$

[5] By the way, this could be a rationale for solving the problem posed by financial convertibles, as we have remarked in footnote 3. In this particular example, and when transaction costs are not an obstacle, a contingency fund may be designed by means of a sinking fund. If conversion were brought into completion before the maturity of the sinking fund, at the latter date an alternative allocation of the capital could be found eventually. However, and all along this paper, we deal with financials without convertibility properties.

[6] The reader is referred to Appendix 1 for a direct proof of (1), and to Appendix 2 for a general proof using the Principle of Mathematical Induction.

Whereas this straightforward model has been widely taught and drilled in university courses, so becoming a truly useful device that makes students conversant with capital formation processes, it runs into trouble any time we attempt to implement it in real settings. It's worth taking this matter further.

If investors[7] or companies intended to follow this model outright, they should commit **C** dollars at the beginning of each period. Furthermore, returns to be expected from cash flows would be explained by the rate of interest **I** that this model assumes to be constant, a rather implausible target to achieve in real life[8].

Among workable ways to tie up a contractual constant rate, two of them have currently been employed:

a) The company or investor enters into a long-term agreement with some financial institution that could grant the capital formation in accordance with the program[9] conveyed by relationship (1). It goes without saying that in such environment the financial institution should hedge the rate-of-interest risk on its own, usually by means of financial derivatives. This kind of transaction conventionally takes place in the money market and, most of the time is carried out by means of chartered banks.

b) Otherwise, the company or investor might get access to a string of forward contracts on rates of interest. Nevertheless, a direct bargain seems unlikely without a broker o dealer that could agree to lock up the same rate of interest along the intended horizon, by means of a combo of derivatives, like a swap of rates of interest, to assist in the development of capital formation. This transaction makes its way through the derivatives market with the assistance of certified brokers or dealers.

For all practical purposes, both procedures stop short of being helpful. Among the main reasons for this to happen we should highlight

[7] For the sake of example, individuals or household investing in real estate or retirement plans. Next, we should include institutional investors, like insurance companies or pension funds, for which the capital formation processes are of the essence, to say the least.

[8] In point of fact, as long as interest rates stayed smooth, banks would be ready to lend by charging fixed rates, or bond issuers would be offering fixed-rate coupons. But since the 70s the staging has undergone deep changes, and the temporal structure of rates of return turned out to be more volatile than ever before.

[9] More often than not we know in advance which is **FV(0; N)**, while **N** and **I** are currently inputs for the program. Hence, we derive the value of **C** from (1) outright.

financial costs faced by companies, and the understandable reluctance on the part of banks and brokers to commit themselves with such transactions instead of undertaking the role of portfolio managers on their own.

Thereupon, we must weigh up the pros and cons of portfolio building as a more tenable methodology.

1.2 PORTFOLIO BUILDING

This approach rests upon the following premises:

a) at the beginning of each period and all along the chosen horizon **H = [0; N]**, the company earmarks cash flows

$$\mathbf{CF\ (\ k\ ) \quad ; \quad k\ =\ 0, 1, 2, \ldots, N - 1}$$

to be allocated to the purchase of financial assets;

b) those assets are put together into a portfolio to provide a sound return so that, at maturity date **N**, a terminal value equal to

$$\mathbf{FV\ (0; N)}$$

might be successfully established;

c) The management of such a portfolio may be handed over to the company's treasurer or, still better, committed to the expertise of an appointed fiduciary agent. In this paper, we definitely embrace the latter alternative.

For the sake of illustration, we are going to bring forward two decision-making procedures that actually meet the former constraints. The first one takes advantage of zero-coupon bonds, whereas the second profits from mainstream tools in the practice of portfolio management.

i) The case for zero-coupon bonds

At the beginning of each period, the company, or the fiduciary agent, purchases a zero-coupon bond that costs

$$\mathbf{PVB(k; N) \quad ; \quad k = 0, 1, 2, \ldots\ldots, N - 1}$$

which will deliver, at maturity date **N**, a monetary amount equal to **B(k; N)** [10].

To put the whole chain of transactions into a sensible framework of analysis, two requirements come in handy:

- a boundary condition that must hold at maturity date:

$$\mathbf{FV\ (0;\ N)} = \tag{2}$$

$$= \mathbf{B(0;\ N) + B(1;\ N) + B(2;\ N) + \ldots + B(N\text{-}1;\ N)}$$

that is to say, zero-coupon bonds end up delivering the targeted amount of money at maturity;

- a rule for buying zero-coupon bonds for (2) to hold eventually; for instance by choosing

$$\mathbf{B(k;\ N) = FV(0;\ N)\ /\ N}$$

It follows from this procedure that we are not longer constrained to cope with constant cash flows and fixed rates. Period after period, market returns and prices will tell the company, or the fiduciary, how much money ought to be disbursed for the purchasing of each zero-coupon.

But a model worked out from zero-coupon bonds gives rise to foreseeable troubles, among which the following are frequent:

- they are not available in developing markets to the same extent they are in New York or London markets; even in global markets, to overcome this drawback by means of Strips from standing Treasury Bills and Bonds is easier said than done;

- sometimes, the buying schedule entails a mismatch between the maturity date of Strips with the maturity claimed by the capital formation process[11];

[10] In the latter scenario, the company sends an amount of money, **PVB(k; N)** dollars, to the fiduciary who manages the sinking fund portfolio.

[11] Albeit such would be a minor issue eventually, provided we could reframe our horizon to match the schedule of available zero-coupons.

- whenever the temporal structure of financial returns followed a persistently declining pattern, to get zero-coupons would become expensive.

Briefly stated, this technique seems appealing enough, but only when the market offers companies and investors with a wide variety of zero-coupons. Unless such supply could be warranted, another sort of engineering would be worth being tried.

ii) Mainstream portfolio management

In this environment we assume that a qualified portfolio manager, who acts as a fiduciary agent[12], will demand the company to deliver, at the beginning of each period, an amount of cash flows equal to

$$\mathbf{CF\ (\ k\ )\quad ;\ k\ =\ 0,\ 1,\ 2,\ \ldots\ ,\ N\text{-}1}$$

The money will be applied to the purchase of financial assets with the ultimate goal of building up a portfolio **P** to be held till maturity at date **N**, earning returns period after period to raise a future capital equivalent to **FV(0; N)** dollars in the end.

Whereas such approach is wide-ranging in scope and gathers the blessings of practitioners, implementation brings up four problems to be solved:

a) Who is to manage the portfolio and up to what costs?

b) By which means the investors' rights are kept safe?

c) How to determine the most fitting pattern of cash flows for the capital formation process?

d) How to deal with the portfolio's underperformance?[13]

At this juncture, we turn to question c). The alternative I am advocating here is not the only one available, but it adds weight to our governance framework of analysis. An advantage of this proposal lies

[12] It should be noted that such agent owes fiduciary duties to the company's creditors. In other cases, like personal investments, or acquisition of fixed assets by a company, the agent will be fiduciary of the investor or the company.

[13] The first two questions are of paramount importance since answering them leads to the role of a fiduciary agent, a topic that will be settled down in subsection 2.2. It will be for subsection 1.3 to deal with the problem of underperformance.

on the fact that keeps the standard model as a benchmark and furnishes practitioners with a simple course of action to redress likely imbalances in the portfolio.

Basically, it comes down to a string of quasi-variable cash flows, **CF(k)**, that can be split down into two components:

$$\mathbf{CF\,(\,k\,) \quad = \quad C \; + \; G(\,k\,)} \tag{3}$$

where **C** stands for the constant amount of money we should allocate at the beginning of each period, in the event that capital formation would be tied together with a fixed reinvestment rate **I**. To put it another way, **C** is the cash flow that would be determined if we used the standard model[14].

On the other hand, **G(t)** measures up the variable amount of money required to fill the gap between what the manager should have got if reinvestment had been earning the rate **I**, that is

$$\mathbf{FV(0;\,k)}$$

and the actual value of the portfolio

$$\mathbf{V_{P}\,(\,k\,)}$$

that had been accruing along the period **(k–1; k)** out of the financial assets held in the portfolio **P** till date **k**.

The message conveyed by (3) can be briefed as follows:

> *At date **k**, the company or the investor must provide the manager with **C** dollars for the new period, **(k; k+1)**, plus the gap in value, **G(k)**, to be added to the portfolio **P** at that date, in case that the return of period **(k – 1; k)** would have been less than the rate of interest **I**.*
>
> *If the portfolio had outperformed the target value, then **G(k)** would have become a credit to the company or investor. Hence, it carries a negative value in (3).*

Therefore, we get hold of a hard-and-fast decision rule:

---

[14] Background of this model was given in section 1.1, while a rigorous treatment can be found in Appendices 1 and 2.

- If $\mathbf{V_P(k) < FV(0;k)} \Rightarrow \mathbf{G(k) > 0}$

then the company must deliver more than **C** dollars to the fiduciary, at date **k**.

- If $\mathbf{V_P(k) > FV(0;k)} \Rightarrow \mathbf{G(k) < 0}$

then the company must deliver less than **C** dollars to the fiduciary, at date **k**.

Remark:

Why should we stick to a capital formation process that yields a constant rate of interest when all is said and done, while a portfolio manager could attain higher returns following a more active strategy?

*Compliance risk is the answer. The fiduciary agent builds up **FV(0;N)** dollars and he shouldn't seek for hazardous risk-return profiles[15], concerned lest he failed to comply with the fiduciary duties of loyalty, diligence and disclosure, as we are going to highlight in section 2.2.*

Being capital formation a step-by-step development, we have to keep a record about how gap-fillers **G(k)** are threaded among them as we shift from one period to the next one. And this will be the topic for next subsection.

1.3 HOW TO SOLVE THE RESETTING ISSUE

The resetting issue calls forth two consequential commitments that are by no means easy to disentangle:

a) it must submit an algorithm to work out the amount of dollars we need to fill the gap that stems from the portfolio value at the end of each period;

b) it lays the foundations for appointing a fiduciary agent to manage the portfolio, so that transparency and accountability could be safeguarded.

[15] The whole issue hinges upon better governance. We have coped with this matter in *Tailoring Compliance Risk and the Compliance Function for Non-Financial Organizations* (Apreda, 2006, also in 2007d).

Whereas the fiduciary feature will be expanded on further in subsection 2.2, this is the place where we are going to track down how the portfolio should be retooled at the beginning of each period[16].

Therefore, we will work out an iterative program that proceeds by stages, each of them consisting of rules that bring about certain results out of inputs coming along from earlier stages. The format requires the following entries[17]:

- starting value;
- starting resetting value for the portfolio;
- final value, as expected by using the standard model;
- final value of the portfolio;
- assessing the gap in return;
- assessing the gap in value;
- final resetting value for the portfolio, which becomes the starting value to be used in next iteration.

Stage 1, ending at date t = 1

starting value:

$$\mathbf{FV(0;\,0) \quad = \quad C \quad = \quad V_P(\,0\,)} \tag{S1-1}$$

final value, expected for the end of this stage (t = 1):

$$\mathbf{FV(0;\,1) \quad = \quad FV(0;\,0)\,.\,[1 + I\,]} \tag{S1-2}$$

final value, realized at the end of this stage (t = 1):

$$\mathbf{V_P(1) \quad = \quad V_P(\,0\,)\,.\,[1 + R(0;\,1)\,]} \tag{S1-3}$$

assessing the gap in return:

$$\mathbf{[1 + I\,] \; = \; [1 + R(0;\,1)]\,.\,[1 + g(0;\,1)]} \tag{S1-4}$$

*assessing the gap in value***:**

we plug (S1-1), (S1-3) and (S1-4) into (S1-2):

[16] The iterative numerical program will be run through subsequent stages, leaving for Appendix 3 the inductive proof of the algorithm.

[17] At stage **k** we will need all of the entries; at stages 1 and 2 some of them can be skipped.

$$\mathbf{FV(0;1) = FV(0;0)\,.\,[1 + I\,]}$$

$$\mathbf{FV(0;1) = V_P(0)\,.\,[\,1 + R(0;1)]\,.\,[1 + g(0;1)]}$$

which leads to

$$\mathbf{FV(0;1) = V_P(1)\,.\,[\,1 + g(0;1)\,]} \tag{S1-5}$$

or, equivalently,

$$\mathbf{FV(0;1) = V_P(1) + V_P(1)\,.\,g(0;1)}$$

where

$$\mathbf{V_P(1)\,.\,g(0;1)}$$

measures the gap in value at the end of the first period. In other words,

$$\mathbf{G(1) = V_P(1)\,.\,g(0;1)}$$

the rate **g(0; 1)** being positive whenever the portfolio comes short of its target, and negative otherwise.

Stage 2, ending at date t = 2

starting value: taking advantage of (S1-5)

$$\mathbf{FV(0;1) + C = V_P(1)\,.\,[1 + g(0;1)] + C} \tag{S2-1}$$

resetting value for the portfolio at the start of this stage:

$$\mathbf{V^*_P(1) = V_P(1)\,.\,[1 + g(0;1)] + C} \tag{S2-2}$$

final value, expected for the end of this stage (t = 2):

$$\mathbf{FV(0;2) = < FV(0;1) + C >\,.\,[1 + I\,]} \tag{S2-3}$$

final value, realized at the end of this stage (t = 2):

$$\mathbf{V_P(2) = V^*_P(1)\,.\,[1 + R(1;2)\,]} \tag{S2-4}$$

assessing the gap in return:

$$\mathbf{[\,1 + I\,] = [\,1 + R(1;2)\,]\,.\,[\,1 + g(1;2)\,]} \tag{S2-5}$$

*assessing the gap in value***:**

by plugging (S2-1), (S2-2), (S2-4) and (S2-5) into (S2-3), we get

$$\mathbf{FV(0;\,2) \;=\; <\; FV(0;\,1) \;+ C \;>\, .\; [\; 1 \;+\; I\;]}$$

$$\mathbf{FV(0;\,2) \;=\; V\,*_{P}(\,1\,)\;.\;[\;1\;+\;R(1;\,2)\;]\;.\;[\;1\;+\;g(1;\,2)\;]}$$

which amounts to

(S2-6)

$$\mathbf{FV(0;\,2) \;=\; V_{P}(2)\;.\;[1 +\; g(1;\,2)]}$$

or, likewise,

$$\mathbf{FV(0;\,2) \;=\; V_{P}(\,2\,) \;+\; V_{P}(\,2\,)\;.\;g(1;\,2)}$$

and the gap in value finally becomes

$$\mathbf{G(2) \;=\; V_{P}(\,2\,)\;.\;g(1;\,2)}$$

Stage k, ending at date t = k

Former stages have uncovered the pattern of an iterative algorithm whose derivation will be rendered in Appendix 3. Hence, for any arbitrary stage **k** the following relationships will hold:

starting value:

(S3-1)

$$\mathbf{FV(0;\,k-1) \;+\; C \;=}$$

$$\mathbf{=\;\; V_{P}(\,k-1)\;.\;[\;1\;+\;g(k-2;\,k-1)\;]\;+\;C}$$

resetting value for the portfolio at the start of this stage:

(S3-2)

$$\mathbf{V\,*_{P}(\,k-1) \;\;=}$$

$$\mathbf{=\;\; V_{P}(\,k-1)\;.\;[\;1\;+\;g(k-2;\,k-1)\;]\;+\;C}$$

final value, expected for the end of this stage (t = k):

(S3-3)

$$\mathbf{FV(0;\,k) = <FV(0;\,k-1) \;+ C >\,.\,[1 + I\,]}$$

final value, realized at the end of this stage (t = k):

(S3-4)

$$\mathbf{V_{P}(\,k\,) \;\;=\;\; V\,*_{P}(\,k-1)\;.\;[\;1\;+\;R(k-1;\,k)\;]}$$

assessing the gap in return:

$$\mathbf{[\ 1\ +\ I\ ]\ =\ [\ 1\ +\ R(k-1;k)\ ]\ .\ [\ 1\ +\ g(k-1;k)\ ]} \tag{S3-5}$$

assessing the gap in value:

by plugging (S3-1), (S3-2), (S3-4) and (S3-5) into (S3-3), we get

$$\mathbf{FV(0;k)\ =\ <\ FV(0;k-1)\ +C\ >\ .\ [\ 1\ +\ I\ ]}$$

and, subsequently

$$\mathbf{FV(0;k)\ =}$$

$$\mathbf{=\ V*_P(\ k-1)\ .\ [\ 1\ +\ R(k-1;k)\ ]\ .\ [\ 1\ +\ g(k-1;k)\ ]}$$

hence:

$$\mathbf{FV(0;k)\ =\ V_P(\ k\ )\ .\ [\ 1\ +\ g(k-1;k)\ ]} \tag{S3-6}$$

or, likewise,

$$\mathbf{FV(0;k)\ =\ V_P(\ k\ )\ +\ V_P(\ k\ )\ .\ g(k-1;k)}$$

and the gap in value becomes

$$\mathbf{G(k)\ =\ V_P(\ k\ )\ .\ g(k-1;k)} \tag{S3-7}$$

final resetting value for the portfolio at the end of this stage (t = k):

$$\mathbf{V*_P(\ k)\ =\ V_P(\ k\ )\ .\ [\ 1\ +\ g(k-1;k)\ ]\ +\ C} \tag{S3-8}$$

It goes without saying that the quasi-variable cash flow **CF(k)**, already defined in (3), can be worked out from (S3-7) and (S3-8):

$$\mathbf{CF(\ k\ )}\ =\ \mathbf{V_P(\ k\ )\ .\ g(k-1;k)\ +\ C}$$

2. THE PRAGMATICS OF SINKING FUNDS

Broadly speaking, by **Pragmatics** we are to understand the study of decision-making and problem-solving issues, by means of down-toearth, empirical, learning-by-doing, and experimental approaches[18]. All over this paper, the expression will stand for the practical and purposeful usage of sinking funds in business strategies[19].

With a view to illustrating sinking funds in actual practice, we are going to point up the following examples:

- from the side of a single investor, a sinking fund becomes functional in capital formation processes related to real estate, an insurance package, a retirement plan to meet old age, or to attend future education expenses for the family's children;

- institutional investors (like pension funds and insurance companies) currently use them as vehicles to planning for capital needs ahead in the future;

- among companies in the private sector, sinking funds come in handy for a variety of purposes, namely:

– replacement of standing capital assets as well as the purchase of new ones;

– the framing of investment decisions in the future[20];

---

[18] Pragmatic actions, procedures, beliefs and knowledge, became the subject matter of many contributions made along the history of ideas. Conspicuously, **Pragmatism** stands out among them and can be regarded as the philosophical attempt to explain epistemological issues, in particular the theory of knowledge and the methodology to be applied in scientific research, by stressing practical grounds and avoiding dogmatic standpoints. In the words of William James (1907):

The whole function of philosophy ought to be to find out what definite difference it will make to you and to me, at definite instances in our life, if this world-formula or that world-formula be the true one.

Pragmatism as a philosophy for human action was developed by a group of distinguished American scholars, among which we find Charles Sanders Pierce, William James, John Dewey, and the famous Judge Oliver Wendell Holmes. Background on their lives and work, as well as how they influenced American culture can be tracked down in a highly remarkable book *The Metaphysical Club*, written by Louis Menard (2001). A classical rendering of this philosophical movement is *Pragmatism*, by William James (1907,1990).

[19] In the field of Linguistics, the word "pragmatics" is widely used. It refers to how we deal with words and expressions by attaching them distinctive meanings when we interact with other people. On this account, see for example the *Oxford Dictionary of English Grammar*, by Sylvia Chalker and Edmund Weiner (OUP, 1994, Oxford, UK).

[20] As it would be with the engineering for real options or, by the way, the so-called staging finance in venture capital endeavors.

– the repayment of debt to bondholders and institutional investors;

– by the same token, the paying back of bank liabilities;

– redemption of financial hybrids, like preferred stock, convertible preferred stock with an in-built maturity feature[21], bonds with Warrants, dual preferred stock and debt, convertible bonds;

– incentive packages to senior management and directors[22].

The pragmatic viewpoint that I am advocating in this paper intends to handle two subject matters often constrained by transactional and regulatory environments:

i. the choice of a mechanism for capital formation;

ii. the fiduciary role performed by an appointed portfolio manager.

2.1 THE CHOICE PROBLEM

Let us assume we are interested in designing a mechanism for paying back the principal of a bond to be issued shortly. Which would be the most fitting alternatives at hand?

a) Firstly, we could float a bullet bond, which pays the full amount of principal at maturity date **N**. For certain, it is a widespread style of repayment in the corporate world. However, if the company does not build up a sinking fund, as maturity approaches it must find out how to meet the liability, either by issuing new securities or depleting its own savings[23]. By and

[21] Standard preferred stock, like ordinary shares, does not convey any definite maturity date; in point of fact, they have an indefinite span of life. In contrast, lately designs for preferred stock endow them with maturity, for instance dual preferreds (principal redeemed in one currency, while preferred dividends in other) or convertible preferred (for which a mandatory maturity date is essential for the conversion mechanism to end up outright). More on this issue can be found in footnotes 3 and 5.

[22] Background on preferred stock can be found in Apreda (2004) that focuses on one- and two-tiered convertible preferred stock with the purpose of meeting debt restructuring, the enhancing of the capital structure, as well as the design of compensation packages.

[23] This may be accomplished, for instance, through retained earnings (a rather debatable decision, by all means), or by selling financial assets that have been kept in the company's portfolio of financial investments (a much more pragmatic stance).

large, as maturity date nears, risky strains may be put on the company's Treasury.

b) Another choice may consist in issuing a bond that makes a string of consecutive repayments so as to cancel the principal progressively along its life expectancy. By far, it is a well-known procedure, even used by governments when placing sovereign bonds in the market. Among its advantages, it can be said that allows the company's Treasury to deal with the liability at a more relaxing pace than in the case of bullets. On the other hand, the downside lies on the fact that many markets as well as a variegated range of investment groups would rather avoid them on account of either taxes arising out of partial repayments, or diminishing interest payments.

c) There are also bonds that ultimately blend features outlined in a) and b). For the sake of example, let us assume the company issues a bullet with an in-built contingency by which as from certain date, it keeps the right open to repurchase the standing bond (the oft-quoted "call provision"). For example, the company's Treasury could find sensible to trade off high levels of liquidity against declining levels of rates of interest. But such a device could become a stumbling block for investors (or certain market preferred habitats). For one thing, a call provision prevents investors from collecting interest payments as from the exercise of such an option. For another, they would likely request from the company a counterprovision so as to exercise the right to sell the security back to the issuer (the so-called "put provision").

d) Finally, we may also weigh up the choice of issuing bonds with a a sinking fund. Needless to say that if any company picked up one of the financial vehicles reviewed above, then the problem would be settled outright, doing without any sinking fund.

The four paths of action reviewed above stand as examples that illustrate, for all intents and purposes, what should be meant under the heading of "**the choice problem**".

2.2 THE FIDUCIARY AGENT

Once the company reaches the decision in favor of a sinking fund, bondholders and banks would play on their safest side by claiming greater latitude in monitoring how the company designs and handles the vehicle along the planning horizon until maturity.

To get rid of pervading weaknesses lying around any sinking fund internally managed, there has been a distinctive shift in global financial markets towards appointing an external fiduciary agent to oversee the capital formation process[24].

As it was developed in section 2.1, our proposal hinges upon the fact that the company will budget cash flows, according to relation (3)

$$\mathbf{CF(k) = C + G(k)}$$

to be disbursed at the beginning of each period, turning over them to the manager, who will purchase financial assets and hold the ensuing portfolio in trust till maturity.

The fiduciary agent must comply with distinctive duties and tasks. Besides, appointing such agent also entails costs that will be ultimately paid by the company[25]. Let us move on to each of them in turn.

- ***Duties***

Currently, two essential duties are highlighted in the framework of a fiduciary relationship: **care and loyalty**. However, boundaries between these duties overlap to some extent, as Easterbrook and Fischel (1991) stressed when dealing with this topic in their influential book,

"ultimately, though, there is no sharp line between the duty of care and the duty of loyalty".

The **duty of care** stems from the need of acting "as a prudent person does in the management of his own affairs of equal gravity"[26], which stands for diligence. The **duty of loyalty** assumes a behavior that

[24] Lawrence Wilsey (1947), after doing an extensive research of preferred stock in the United States markets, pointed up that many companies after the Second World War gave up the management of their own sinking funds on behalf of a fiduciary agent.
[25] I acknowledge here a helpful remark raised by Professor Marcelo Villegas (Ucema), who suggested me to factor in fiduciary costs in this section.
[26] Easterbrook and Fischel, chapter 4, p. 103.

pursues, facing any conflict of interest, the fulfillment of the interests of principals instead of the fiduciary's own agenda[27].

For the last decade some scholars and practitioners from the field of Corporate Governance [Black (2001), for instance] have been laying emphasis on the convenience of adding the **duty of disclosure** either in open or closed companies, and we could not agree more with such proposal. This duty points to full disclosure of information and it borrows from the more constrained **duty of candor**, which is focused on the disclosure of material information[28]. In the case of open companies, it stresses disclosure to shareholders before they vote, but also requests Directors to diligently oversee self-dealing transactions.

- ***Tasks***

The fiduciary agent must undertake a complex set of activities in compliance with its duties. To name but a few:

a) Monitoring that the company actually submits contractual cash flows, comprising those arising from the resetting of the portfolio value and interest payments owed to creditors.

b) At maturity date, the sinking fund will have brought the principal to completion, and the fiduciary agent must pay it off to bondholders.

c) In case the company had pledged real assets as collateral, the fiduciary agent must audit that no misappropriation would have taken place during the bond life.

d) Purchasing financial assets to build up the portfolio that he must manage.

e) Periodically, to disclose information both relevant and material to the creditors' interests.

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[27] In particular, conflicts arising from self-dealing transactions. As Professor Bernard Black (2001) remarked, "*in the United States, the duty of care is mostly an aspirational statement about how directors should try to act, and not a basis for liability if the directors fall short of this standard*".

[28] Although "material" is a catchword, there is a meaning that has proved suitable for corporate governance. Following the Black's Law Dictionary, by material is currently understood *something of such nature that knowledge of the item would affect a person's decision-making process*. It goes without saying that lack of consequential information may bring about either monetary or reputational losses, as well as regulatory punishments.

f) To perform as a broker of asymmetric information between the company and its bondholders, so as to safeguard creditors' rights against any abuse or misdeed[29].

- ***Costs***

When any company sticks to a sinking fund being externally managed, it must bear the costs since creditors are not expected to foot the bill. On the other hand, these costs are to be traded off against likely rewards, either intangible or tangible ones:

- among **intangible benefits** we can point to the reputational signal that spreads over capital markets after announcing the preference for a sinking-fund vehicle;

- by far, the most relevant **tangible benefit** lies on the extent to which the coupon-rate of interest decreases in relation to the level offered by a similar security without the sinking fund provision.

Whereas costs may be assessed as a matter of course, rewards are rather elusive to deal with. Nevertheless, and taking a further step in this line of argument, let us denote the estimated rate of interest offered by the financial when it is devoid of any sinking fund

$$\mathbf{R(no\ sinking\ fund)}$$

while

$$\mathbf{R(sinking\ fund)\ ,\quad C(fiduciary\ agent)\ ,\quad \Delta R(credit\text{-}risk\ rating)}$$

will stand for, respectively, the rate of interest offered when a sinking fund is attached to the bond, the cost rate of appointing the fiduciary agent, and the expected incremental jump of the credit-risk rating in the aftermath of the bond placement into the market.

We can contrive a break-even point relationship to link all these rates, working out which would be the greatest coupon rate

$$\mathbf{R(sinking\ fund)}$$

for the bond to contest another one with no sinking fund. In point of fact, the break-even rate stems from the following relationship:

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[29] The use of corporate governance brokers and how they trade in asymmetric information was developed for the first time in Apreda (2007c).

$$[\,\mathbf{1 + R(no\ sinking\ fund)}\,] \;=\; [\,\mathbf{1 + R(sinking\ fund)}\,]\;\mathbf{.}\tag{4}$$

$$\mathbf{.}\;[\,\mathbf{1 + C(fiduciary\ agent)}\,]\;\mathbf{.}\;[\,\mathbf{1 + \Delta R(credit\text{-}risk\ rating)}\,]$$

The reader should bear in mind that credit risk rating would likely improve with the sinking fund, and hence there should follow a cutback in the risk measure, which leads **ΔR(credit-risk rating)** to ultimately carry a negative sign.

Therefore, next inequality

$$[\,\mathbf{1 + R(no\ sinking\ fund)}\,] \;>\; [\,\mathbf{1 + R(sinking\ fund)}\,]\;\mathbf{.}\tag{5}$$

$$\mathbf{.}\;[\,\mathbf{1 + C(fiduciary\ agent)}\,]\;\mathbf{.}\;[\,\mathbf{1 + \Delta R(credit\text{-}risk\ rating)}\,]$$

becomes a sufficient condition to make the sinking fund fully tenable in the end.

What (5) brings to light is the fact that a bond with sinking fund can actually pay a lower interest rate than another bond without such feature. The size of such rate involves a trade-off between transaction costs and credit-risk ratings. The compounding of these three variables (coupon interest, credit-risk rating and transaction costs) should be less than the size of the coupon rate offered by the bond without sinking fund.

It goes without saying that relationship (5) is one among other suitable benchmarks to establish the convenience or not of adopting a sinking fund. On this regard, great care is needed in gaining knowledge of intangible features that contribute to the governance of a company when this mechanism is set up, although their assessment could be not so easily tractable like credit ratings. Next section will expand on how it is possible to get hold of intangibles that foster governance.

3. COVENANTS AND GOVERNANCE

How could we actually upgrade the company's governance after engaging a fiduciary agent? This may be mainly accomplished through four courses of action:

a) The company should embed a sinking-fund provision in the Charter of the organization or, still better, in the Governance Statute, as I have recently set forth elsewhere[30]. It must be a sort of multi-purpose provision that comprises capital formation processes intended for equity, debt and financial hybrids issues, as well as capital assets replacement.

b) The appointment of a fiduciary agent fosters not only accountability but compliance as well. Firstly, because the fiduciary holds the company accountable and abiding by the covenants attached to the sinking fund. Secondly, since investors can follow up the extent to which both the company and the fiduciary agent meet their contractual duties. Simultaneously, transparency will improve as long as the fiduciary is empowered to request from the company full disclosure of all information that could be consequential for the interests of investors and banks.

c) Putting up capital with the help of a sinking fund brings pressure at least in three governance-related strands[31]:

- owners as well as the Board of Directors remain safe in the knowledge that managers would be prevented from wheeling and dealing with budgeted cash flows;

- the sinking fund, when managed externally, forestalls further rent-seeking, soft budget constraint, and tunneling[32];

- by and large, the company is able to exhibit and carry out better corporate practices towards creditors and stockholders.

d) If the sinking fund were employed in coping with the replacement of fixed assets, or with a new investment decision, the fiduciary would

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[30] The Governance Statute gathers those distinctive principles of Corporate Governance that bind up the company with stakeholders and transactional environments. More background on this issue in Apreda (2007a).

[31] It's worth bearing in mind that breach of debt covenants eventually triggers off a default environment.

[32] Tunneling is a recently coined expression, of which more background and references can be found in Apreda (2007b). Broadly speaking, the expression stands for processes and activities by which resources and property rights are shifted illegally, opportunistically and with guile, from one company to another in the same group (or along different divisions in the same company). The transactions involved take place by means of financial or accounting "tunnels" that remain out of sight without being held accountable to other stakeholders and regulators.

become a gatekeeper who oversees the whole process, giving rise to stark upsides in governance, namely:

i. asset management within the company is sharpened up;

ii. the Board of Directors stands up for its own fiduciary duties in monitoring what happens with the company's assets in the quest of corporate value enhancement;

iii. when depreciation charges are managed by a fiduciary fund, they do not contribute any longer to the company's self-financing[33]. This outcome has two overlapping readings:

- *self-financing is narrowed down to its natural sources, that is to say retained earnings and the portfolio of financial assets looked after by the company's Treasury;*

- *sinking funds avoid any misuse of depreciation charges under the guise of free-cash-flows, frequently leading to wheeling and dealing, empire-building, perks-consumption, or tunneling.*

3.1 DESIGNING A PROTOCOL FOR SINKING FUNDS

For the sake of illustration, we are going to outline a protocol[34] by listing ground rules to follow when a sinking fund is attached to a bond placement[35].

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[33] A remark is due here on account of the depreciation method to be used. The standard model for capital formation requires a fixed depreciation charge

Dep (k) = C

at the beginning of each period. But, in contradistinction with the standard model of capital formation, periodical allocations of **Dep (k)** dollars would be preferred when taking place at the end of each period. On the other hand, if **Dep(k)** ought to be variable (and this is contingent upon the chosen depreciation schedule) an iterative process like the one outlined in subsection 1.3 ought to be designed eventually.

[34] "Protocol", in this context, stands for a set of formal rules of behavior and basic agreements among creditors, the company and the fiduciary agent. Hence, the protocol must be regarded as a basic covenant that is actually attached to the bond's prospectus.

[35] By the same token, a similar protocol could be drafted in the case of acquisition or replacement of fixed assets, on the one hand, as well as issuing financial hybrids, on the other.

i. The issuer of this bond pledges to creditors full compliance of all and every commitment conveyed by the present covenant.

ii. The issuer of this bond must seek for an independent and reputable fiduciary agent who has to give certified evidence of his track record before being contracted out.

iii. The fiduciary agent must employ cash flows delivered to him from the company to purchase financial assets, build up and manage the ensuing portfolio. The financial assets low-risk investments: for instance, Treasury Bonds, Notes and Bills, highly rated corporate zero-coupon bonds, or liquid substitutes like time deposits in banks with the highest credit-risk quality.

iv. The company has to meet a twofold schedule to turn over cash flows to the fiduciary:

   a) payments of interest by their maturity dates;

   b) cash-flows arising from the resetting of the portfolio following a transparent algorithm[36].

v. At maturity date, the sinking fund will have brought the principal to completion, and the fiduciary agent must pay it off to bondholders.

vi. When issuing the bond, the prospect must disclose all relevant and material information on behalf of creditors, either related to the bond itself, the sinking fund's features, or the role of the fiduciary agent.

vii. A reliable and independent risk-rating company must rate not only the bond but also the fiduciary's performance.

viii. Being the fiduciary agent the caretaker of the sinking fund, he has to discharge four complementary tasks:

   - to hold the company accountable for the schedule of disbursements defined in item iv;

   - to keep an updated record of each investor;

[36] For instance, the sort of algorithm introduced in section 1.3.

- to call for an annual meeting of bondholders' representatives;
- to request a fairness opinion from experts when any decision or future transaction might put in danger bondholders' interests.

ix. If real assets had been pledged as collateral for the bond placement, it is a duty for the fiduciary agent to survey the integrity and valuation changes of those assets so as to protect bondholders' rights over them. Covenants upon collateral also refers to:

  - forestalling any decision that could lessen assets' value;

  - rejecting any reorganization process, including mergers and acquisitions, from which any damage to bondholders could ensue;

  - establishing the conditions by which certain covenants could be lifted under exceptional circumstances provided that bondholders agree to give their consent.

x. The company, the fiduciary agent, and creditors will be tied together and held answerable by a multiple-agency contract that might be contested in court by any of the counterparts. Breach of covenants by the company will trigger off default environments.

xi. The company will pay the fiduciary both fees and expenses arising from the discharge of the agent's duties and tasks, inclusive of all closely related transaction costs.

xii. At maturity date, after paying off the principal and bringing to completion any adjustment needed for the settlement of the final balance, the fiduciary fund comes to an end, as well as his fiduciary relationship towards investors. However, this action will be contingent upon a positive **Statement of Compliance** issued by an external and independent auditor.

CONCLUSIONS

It's time to briefly put together the main contributions of this paper:

a) We have giving foundations on how sinking funds may be successfully devised as capital formation processes within the framework of the portfolio management approach.

b) Whenever underperformance arises, the resetting of the portfolio follows next, by means of a very simple algorithm that was introduced in section 1.3.

c) By endorsing the appointment of an outside fiduciary agent to manage the sinking- fund portfolio, we set forth a solution to the choice problem, as well expanded on ways of shaping up the company's governance.

d) Last of all, and to make a case for bond placements with a sinking fund provision, we have come up with a protocol by which bondholders, the company, and the fiduciary agent avail themselves of a sheltered covenant.

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APPENDIX 1
PROOF OF THE STANDARD MODEL

If we earmark cash flows of equal value **C** at the beginning of each period, and they always earn the same rate of interest **I** period after period, then at maturity date **N** the capital formation will amount to:

(A1-1)

$$\mathbf{FV(0;N) = C.[1+I]^{N} + C.[1+I]^{N-1} +}$$

$$\mathbf{+ C.[1+I]^{N-2} + \ldots\ldots\ldots + C.[1+I]^{2} + C.[1+I]^{1}}$$

now we divide (A1-1) by [1 + I] to get

(A1-2)

$$\mathbf{FV(0;N) / [1+I] =}$$

$$\mathbf{= C.[1+I]^{N-1} + C.[1+I]^{N-2} +}$$

$$\mathbf{+ C.[1+I]^{N-3} + \ldots\ldots\ldots + C.[1+I]^{1} + C}$$

substracting (A1-2) from (A1-1):

$$\mathbf{FV(0;N) - FV(0;N) / [1+I] = C.[1+I]^{N} - C}$$

which leads us to

$$\mathbf{I.FV(0;N) = C.[(1+I)^{N} - 1].[1+I]}$$

To sum up, we have[37]:

(A1-3)

$$\mathbf{FV(0;N) = [C/I].[(1+I)^{N} - 1].[1+I]}$$

APPENDIX 2
THE CUMULATIVE VALUE OF THE CAPITAL FORMATION PROCESS

The relationship that solves the problem of the capital formation after **N** periods is quite general. In fact, (A1-3) holds true for every **N**. To sustain this statement, we need to provide an inductive proof.

Stage 1: we have to prove that (A1-3) holds when **N = 1**.

[37] By choosing a pattern of installments at the end of each period, we might have got access to another well-known relationship

$$\mathbf{FV(0;N) = [C/I].[(1+I)^{N} - 1]}$$

$$\mathbf{FV(0;1) = C\,.\,[\,1 + I\,] = [\,C/I\,]\,.\,[\,I\,]\,.\,[\,1 + I\,]}$$

$$\mathbf{FV(0;1) = [\,C/I\,]\,.\,[\,(1 + I)^{1} - 1\,]\,.\,[\,1 + I\,]}$$

Stage 2: if we assume that (A1-3) holds true for **N = k – 1**, then we must prove that also holds for **N = k**.

Therefore, let us imagine that at maturity date **N = k – 1**, the capital formation amounts to

(A2-1)

$$\mathbf{FV(0;k-1) = [\,C/I\,]\,.\,[\,(1 + I)^{k-1} - 1\,]\,.\,[\,1 + I\,]}$$

If we wanted to shift from this date to the following one, we should add a new installment in (A2-1), so that a reinvestment will ensue outright:

$$\mathbf{FV(0;k) =}$$

$$\mathbf{= \{ < [C/I\,]\,.\,[(1 + I)^{k-1} - 1\,]\,.\,[\,1 + I\,] > + C\,\}\,.\,[1 + I\,]}$$

which leads us to the following arrangement:

$$\mathbf{FV(0;k) =}$$

$$\mathbf{C\,.\,[\,1 + I\,]\,.\,\{ < [\,(1 + I)^{k} - (1 + I)\,]\,/\,I > + 1\,\}}$$

and also to

$$\mathbf{FV(0;k) =}$$

$$\mathbf{C.\,[\,1 + I\,]\,.\,< [\,(1 + I)^{k} - (1 + I) + I\,]\,/\,I >}$$

or, equivalently,

$$\mathbf{FV(0;k) = C.\,[\,1 + I\,]\,.\,< [\,(1 + I)^{k} - 1\,]\,/\,I >}$$

whence:

(A2-2)

$$\mathbf{FV(0;k) = [\,C/I\,]\,.\,[\,(1 + I)^{k} - 1\,]\,.\,[\,1 + I\,]}$$

Profitting from relationships (A2-1) and (A2-2), by applying the Principle of Mathematical Induction, it follows that

$$\mathbf{FV(0;N) = [\,C/I\,]\,.\,[\,(1 + I)^{N} - 1\,]\,.\,[\,1 + I\,]}$$

holds true for every **N**.

Remark:

A formal, even elegant, statement of the Principle of Mathematical Induction runs this way:

*Let **G** be a subset of the set of natural numbers, **N**, that is to say:*

$$\mathbf{G} \subseteq \mathbf{N}$$

Assuming that

a) $1 \in \mathbf{G}$

b) if $n \in \mathbf{G} \Rightarrow n + 1 \in \mathbf{G}$

then $\mathbf{G} = \mathbf{N}$.

In actual practice, we usually want to prove that an indexed property, **P(n)**, holds true for every value of n. In other words, **G** comes defined as:

$$\mathbf{G} = \{\, n \in \mathbf{G} \;:\; \mathbf{P(n)} \text{ holds true } \}$$

Details, as well as alternative statements for the Principle of Mathematical Induction, can be found in a highly remarkable book, *Proofs and Fundamentals*, written by Professor Ethan Bloch (2000).

APPENDIX 3
ABOUT THE RESETTING MECHANISM[38]

We want to show that, whatever the value of **N**, the resetting value for the portfolio is given by the iterative process

$$\mathbf{V^*_P(N) = V_P(N) \,.\, [1 + g(N-1; N)] + C} \tag{A3-1}$$

Stage 1: we have to prove that (A3-1) holds when **N = 1**.

Indeed, we have already done it in stage 1, at section 1.3, when we got (S1-5)

$$\mathbf{FV(0; 1) = V_P(1) \,.\, [1 + g(0; 1)]}$$

Next step consisted of setting up the starting valuation conveyed by relationship (S2-1):

---

[38] Bear in mind that most among subsequent expressions will become ex-ante assessments and should be preceded by the expected value operator **E[.]**. It is for the ease of notation that we are going to avoid using the operator, as a matter of course.

$$\mathbf{FV(0;1) + C = V_P(1) . [1 + g(0;1)] + C}$$

from which the resetting value follows outright as

$$\mathbf{V^*_P(1) = V_P(1) . [1 + g(0;1)] + C}$$

Stage 2: if we assume that (A3-1) holds true for **N = k – 1**, then it remains to show that also holds for **N = k**.

If we assume (A3-1) holds true for **N = k – 1** this means that

$$\mathbf{V^*_P(k-1) = V_P(k-1) . [1 + g(k-2;k-1)] + C}$$

The final outcome of the development at the section referred was, precisely, that

$$\mathbf{FV(0;k) = V_P(k) . [1 + g(k-1;k)]}$$

and the only thing we must do is to think about the resetting amount at such date, that must fulfills the equation:

$$\mathbf{FV(0;k)] + C = V_P(k) . [1 + g(k-1;k)] + C}$$

from which it follows that the resetting value is given by

$$\mathbf{V^*_P(k) = V_P(k) . [1 + g(k-1;k)] + C}$$

and by the Principle of Mathematical Induction, (S3-1) holds true for any value of **N**.