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SIR MACRO MODEL: COMPARING THE DECENTRALIZED ECONOMY AND THE OPTIMAL POLICY

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A SIR Macro Model: Comparing the Decentralized Economy and the Optimal Policy.

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Abstract: We present a Simple SIR Macro Model to study the economic impact of an epidemic in a model were agent types are unobservable. We solve for the decentralized economy equilibrium and for optimal solution (subject to the constraint that the planner cannot differentiate between agent types). We find that decentralized economy produces an endogenous lockdown in which economic activity decreases. We find that the decentralized economy will begin its lockdown sooner than what the optimal policy prescribes because agents have an incentive to selfishly avoid infection while waiting for others to get infected, recover and contribute to herd-immunity. The optimal policy in this model is not necessarily about forcing people to stay at home but to force people to get infected early on.

JEL Classification: E1, I1, H0.

Keywords: Epidemic, COVID-19, recessions, lockdown, SIR macro model.

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1) Introduction.

The ongoing COVID-19 pandemic has spurred an impressive amount of research on the effect of an epidemic on the economy. On the Macro SIR[2] Model literature there are two main lines of work. One that looks at optimal policies only, like in Alvarez et al. (2020), Acemoglu et al. (2020) and Garriga et al. (2020). This line of work shows that optimal policies involve lockdowns, but they do not compare the optimal solution to that of a decentralized economy. Garriga et al. compare the optimal solution to what they qualify as an "uncontrolled epidemic" which represents a model in which economic agents do not adjust their behavior in the presence of an epidemic. The other line of research looks at the behavior of the decentralized economy. In this line we find Eichenbaum et al. (2020) and Krueger et al. (2020). Our paper is in line with this branch of the literature.

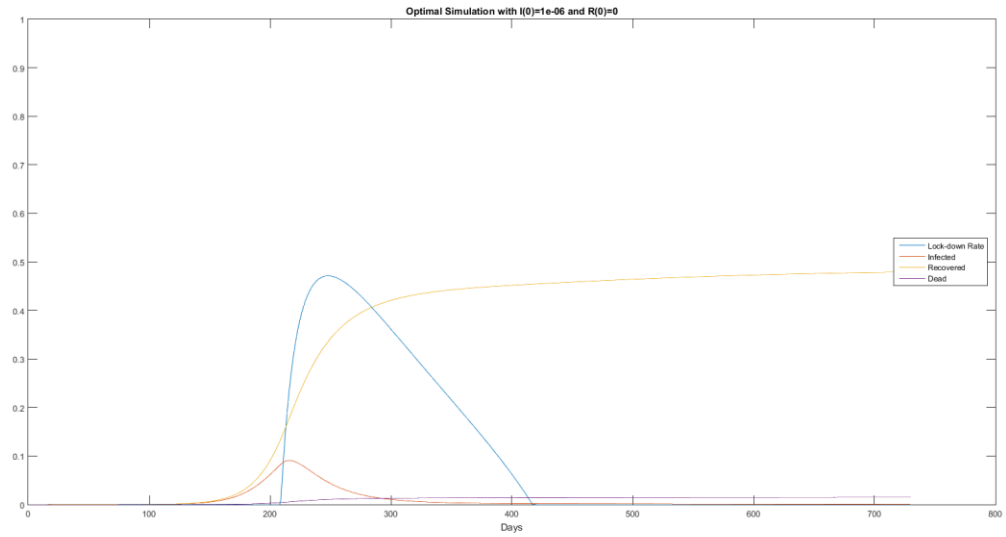
In this paper, we present a simple SIR Macro Model to study the economic impact of an epidemic in a model were agent types are unobservable. We solve for the decentralized economy equilibrium and for optimal solution (subject to the constraint that the planner cannot differentiate between agent types). We find that decentralized economy produces an endogenous lockdown in which economic activity is decreased. We find that the decentralized economy will begin its lockdown sooner than what the optimal policy prescribes because agents have an incentive to selfishly avoid infection while waiting for others to get infected, recover and contribute to herd-immunity. This effect was discussed by Mulligan, Murphy and Topel in their article "Some basic economics of COVID-19 policy" from which I quote

"The gain from reducing the incidence of infections in the presence of externalities may seem obvious, but there are important caveats. First, the externality created by an infected individual can actually benefit others (a positive externality), which would call for less social distancing. Suppression of the disease delays the development of "herd immunity"—if an individual's recovery from the disease creates immunity and so the inability to transmit the disease to others, it might be better if those at low risk (such as the young) were quickly infected, while vulnerable groups were isolated."

The optimal policy in this model is not necessarily about forcing people to stay at home but to force people to get infected early on. This implies that the optimal lockdown policy involves having agents work more than what they would like to when faced with an epidemic. The reason, as suggested by Mulligan et al, is that in this model there are two externalities, one which is negative, sick agents infect healthy one, and another that is positive, agents that recover from the disease become immune and contribute to herd-immunity. At the outbreak of the epidemic the planner lets the disease run free until a sufficiently high number of agents are infected. Once this number is reached the planner implements a lockdown (the magnitude and duration depend on the

[2] SIR stands for Susceptible, Infected and Recovered (or Removed) which refer to the different types of agents epidemiolocal models.

curvature of the production function) to control the spread and wait for herd immunity to be reached. In the decentralized solution agents have an incentive to work less and reduce their probability of becoming infected while waiting for other agents to recover and contribute to herd immunity. This produces a flatter epi curve as shown in the figures 1.a and 1.b.



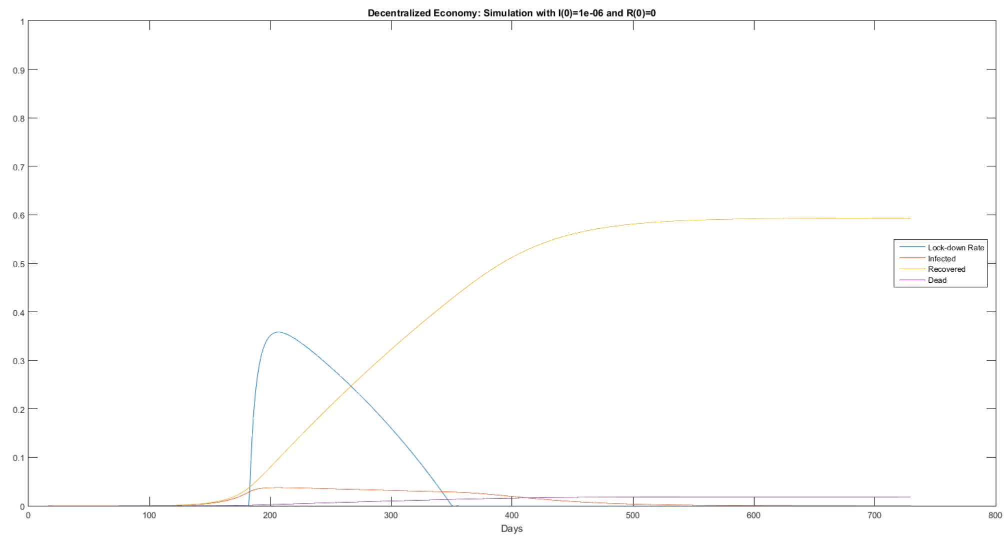


Figure 1. Simulations for an epidemic which starts with 1 infected agent per million people and no recovered agents. Top figure shows the solution to the planner´s problem and bottom figure shows the decentralized economy.

In section 2 we develop the model. In section 3 we solve for the optimal plan and in section 4 we solve for the competitive equilibrium.

2) Model.

There are four types of agents: susceptible, infected, recovered, and dead. The total population is normalized to one

$$N_S(t) + N_I(t) + N_R(t) + N_D(t) = 1$$

Susceptible, infected, and recovered agents are indistinguishable from each other. Each one has $X(t)$ contacts with other agents per period which determines their output (or utility) according to the following production function.

$$F(X) = X^{\alpha}$$

We assume that there is a maximum number of contacts, X_{MAX}, that an agent can have per period. Dead agents have no contacts. Susceptible agents can become infected at a rate σ which depends on the number of contacts and the fraction of infected agents in the overall population

$$\sigma = cX\frac{N_I}{N_S + N_I + N_R}$$

Infected agents leave their state at a rate γ. Once they stop being infected, they either die with probability μ or become recovered agents with probability $(1-\mu)$. Death and recovery are absorbing states. Let $N(t) \equiv N_S(t) + N_I(t) + N_R(t)$ be the number of agents that are alive at time t. These are the laws of motion for the state variables.

$$\dot{N}_S = -\sigma N_S$$

$$\dot{N}_I = \sigma N_S - \gamma N_I$$

$$\dot{N}_R = \gamma(1-\mu)N_I$$

$$\dot{N} = -\dot{N}_D = -\gamma\mu N_I$$

Let $S, I,$ and R be the number susceptible, infected, and recovered agents as a fraction of living agents. The following laws of motion then apply

$$\dot{I} = cXI(1 - I - R) - \gamma I + \delta\mu I^2 \equiv G(X, I, R)$$

$$\dot{R} = \gamma(1-\mu)I + \gamma\mu IR \equiv H(I, R)$$

$$\dot{N} = -\gamma\mu IN$$

We define r0 as the number of agents that an infected agent infects when $R(0) = 0$ and $I(0)$ is close to zero.

$$\text{r0} = \frac{cX_{MAX}}{\delta}$$

3) Planner´s Problem.

In our setup the planner cannot distinguish between different types of agents or it is unable to implement a policy that discriminates among them. Assume that the planner´s problem is to maximize the expected discounted value of output

$$\rho W(I,R,N) = \max_{0<X\leq X_{MAX}} \{F(X) + W_I(I,R,N)G(X,I,R) + W_R(I,R,N)H(I,R) - W_N(I,R,N)\gamma\mu IN\}$$

with

$$\dot{I} = G(X,I,R)$$

$$\dot{R} = H(I,R)$$

$$\dot{N} = -\gamma\mu IN$$

Guess that

$$W(I,R,N) = NV(I,R)$$

then the planner´s problem can be written as

$$(\gamma\mu I + \rho)V(I,R) = \max_{0<X\leq X_{MAX}} \{NF(X) + V_I(I,R)G(X,I,R) + V_R(I,R)H(I,R)\}$$

$$\dot{I} = G(X,I,R)$$

$$\dot{R} = H(I,R)$$

The first order conditions is

$$F_X + V_I G_X \geq 0$$

which holds with equality if $X < X_{MAX}$. The envelope conditions are

$$\gamma\mu V + (\gamma\mu I + \rho)V_I = V_{II}G + V_I G_I + V_{RI}H + V_R H_I$$

$$(\gamma\mu I + \rho)V_R = V_{IR}G + V_I G_R + V_{RR}H + V_R H_R$$

with

$$V = \frac{F + V_I G + V_R H}{\gamma\mu I + \rho}$$

Let $\varphi \equiv V_I$ and $\upsilon \equiv V_R$ so

$$\dot{\varphi} = V_{II}G + V_{IR}H$$

$$\dot{\upsilon} = V_{IR}G + V_{RR}H$$

We can write the optimality conditions as

$$F_X \geq -\varphi G_X$$

$$\dot{\varphi} = \gamma\mu \frac{F + \varphi G + \upsilon H}{\gamma\mu I + \rho} + (\gamma\mu I + \rho - G_I)\varphi - \upsilon H_I$$

$$\dot{\upsilon} = (\gamma\mu I + \rho - H_R)\upsilon - \varphi G_R$$

If $I = 0$ then $X = X_{MAX}$ is a steady state for any value of R. This implies that there is a continuum of steady states. A segment of this continuum can be discarded right away since stability requires that $\frac{\dot{I}}{I} < 0$, i.e.

$$cX_{MAX}(1 - R) > \gamma$$

which implies that

$$R > 1 - \frac{\gamma}{cX_{MAX}}$$

If $cX_{MAX} < \gamma$ then we are in the presence of an disease which is not very contagious and it should disappear on its own without a considerable loss, i.e. it has a r0 < 1 so it is unlikely that it becomes an epidemic. If $cX_{MAX} > \gamma$ then the minimum stable steady state value of R is

$$R_{MIN} = 1 - \frac{\gamma}{cX_{MAX}} = 1 - \frac{1}{\mathrm{r0}}$$

But it could be the case that the optimal solution converges to a lower value of R with $X < X_{MAX}$. This solution, if it exists, should not be interpreted as a steady state but as a limit in which $I_\infty = 0$, $R_\infty < R_{MIN}$ and $X_\infty < X_{MAX}$. As we show later this type of solutions do not exist.

A) Stable Steady States: $I_{SS} = 0$, $R_{SS} > R_{MIN}$ and $X_{SS} = X_{MAX}$.

Setting $\dot{\varphi} = \dot{\upsilon}$ gives

$$0 = \gamma\mu \frac{F + \varphi G + \upsilon H}{\gamma\mu I + \rho} + (\gamma\mu I + \rho - G_I)\varphi - \upsilon H_I$$

$$0 = (\gamma\mu I + \rho - H_R)\upsilon - \varphi G_R$$

$$F_X \geq -\varphi G_X = 0$$

and get

$$\boxed{\varphi_{SS} = \frac{\gamma\mu F_{MAX}}{\rho(G_I - \rho)} < 0}$$

$$\boxed{\upsilon_{SS} = 0}$$

The linear approximation around the steady state is then given by

$$\begin{bmatrix} \dot{\varphi} \\ \dot{I} \\ \dot{R} \\ \dot{\upsilon} \end{bmatrix} = \begin{bmatrix} \rho - G_I & \frac{\gamma\mu\varphi_{SS}G_I}{\rho} - \frac{(\gamma\mu)^2 F}{\rho^2} + (\gamma\mu - G_{II})\varphi_{SS} & -G_{IR}\varphi_{SS} & -H_I \\ 0 & G_I & 0 & 0 \\ 0 & H_I & 0 & 0 \\ 0 & -G_{IR}\varphi_{SS} & 0 & \rho \end{bmatrix} \begin{bmatrix} \varphi - \varphi_{SS} \\ I \\ R - R_{SS} \\ \upsilon \end{bmatrix}$$

This system has eigenvalues $\{G_I, 0, \rho, \rho - G_I\}$. If $R_{SS} > R_{MIN}$ then $G_I < 0$ which implies that the system has one negative eigenvalue and three positive one and it is saddle path stable.

B) Solutions with $I_\infty = 0$, $R_\infty < R_{MIN}$ and $X_\infty < X_{MAX}$.

If $X_\infty < X_{MAX}$ then $F_X = -\varphi G_X$. Let $\omega \equiv \varphi I$, $\delta \equiv \upsilon I$, and

$$Z(X, I, R) \equiv \frac{\dot{I}}{I} = cX(1 - I - R) - \gamma + \gamma\mu I$$

Since the first order condition holds with equality

$$\alpha {X_\infty}^{\alpha-1} = -\omega_\infty c(1 - R_\infty)$$

The growth rates for ω and δ are given by

$$\frac{\dot{\omega}}{\omega} = \gamma\mu \frac{\frac{FI}{\omega} + G + \frac{\delta}{\omega}H}{\gamma\mu I + \rho} + \gamma\mu I + \rho - G_I - \frac{\delta}{\omega}H_I + Z$$

$$\frac{\dot{\delta}}{\delta} = \gamma\mu I + \rho - H_R - \frac{\omega}{\delta}G_R + Z$$

In a steady state they become

$$0 = \rho - G_I - \frac{\delta}{\omega}H_I + Z$$

$$0 = \rho + Z$$

Which imply that

$$X_\infty = \frac{\gamma - \rho}{c(1 - R_\infty)} < X_{MAX}$$

$$\omega_\infty = -\frac{\alpha {X_\infty}^{\alpha-1}}{c(1 - R_\infty)} = -\frac{\alpha {X_\infty}^{\alpha}}{\gamma - \rho}$$

$$\delta_\infty = -\frac{G_I\omega_\infty}{H_I} = -\frac{cX_\infty(1 - R_\infty) - \gamma}{\gamma(1 - \mu) + \gamma\mu R_\infty}\omega_\infty = \frac{\rho}{\gamma(1 - \mu) + \gamma\mu R_\infty}\omega_\infty$$

So $sign(\delta_\infty) = sign(\omega_\infty)$. This is a contradiction since the marginal value of an infected agent is negative while the marginal value of a recovered agent is positive. So the solutions to the planner´s problem has solutions that converge to saddle path stable steady states with $I_{SS} = 0$, $R_{SS} > R_{MIN}$ and $X_{SS} = X_{MAX}$.

We now solve numerically for the saddle paths that end in a stable steady state. We assume the following parameter values

$$\gamma = \frac{1}{14}, \qquad \mu = 0.03, \qquad \rho = \frac{0.04}{365}, \qquad c = 0.0013, \qquad \alpha = 0.4$$

The value of c is chosen such that when the epidemic starts each infected agent infects $\mathrm{r0} = 1.8$ susceptible agents. The first figure shows the saddle paths in the I, R plane for different steady state values of $R_{SS} > R_{MIN} = 0{,}4444$.

Figure 2 shows the saddle paths for different steady states. It is important to check if this figure has saddle paths that cross each other[3]. If it did then we would have to check at each intersection which path has the highest value and choose that one. Figures 3-6 show the value and policy functions from different perspectives. Figure 7 shows the simulation for an economy that starts with $I(0) = 1e - 6$ $R(0) = 0.$ Time is counted in days. The lock down rate is calculated as $1 - \frac{X(t)}{X_{MAX}}$ and the number of infected, recovered, and dead agents are calculated as a fraction of the initial population.

[3] Increasing the value of r0 can produce this result, especially for high values of infected agents.

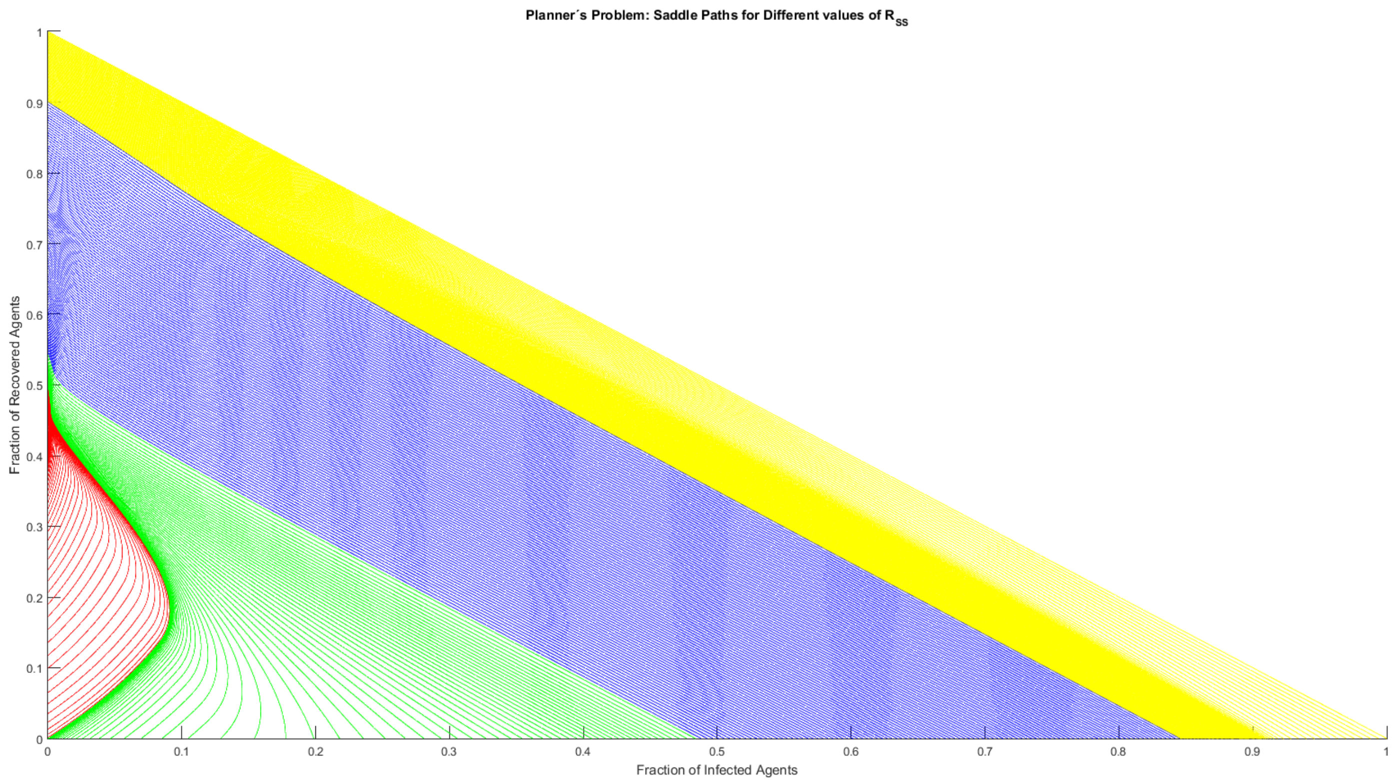


Figure 2: Saddle Paths for the social planner´s problem in the in the I, R plane.

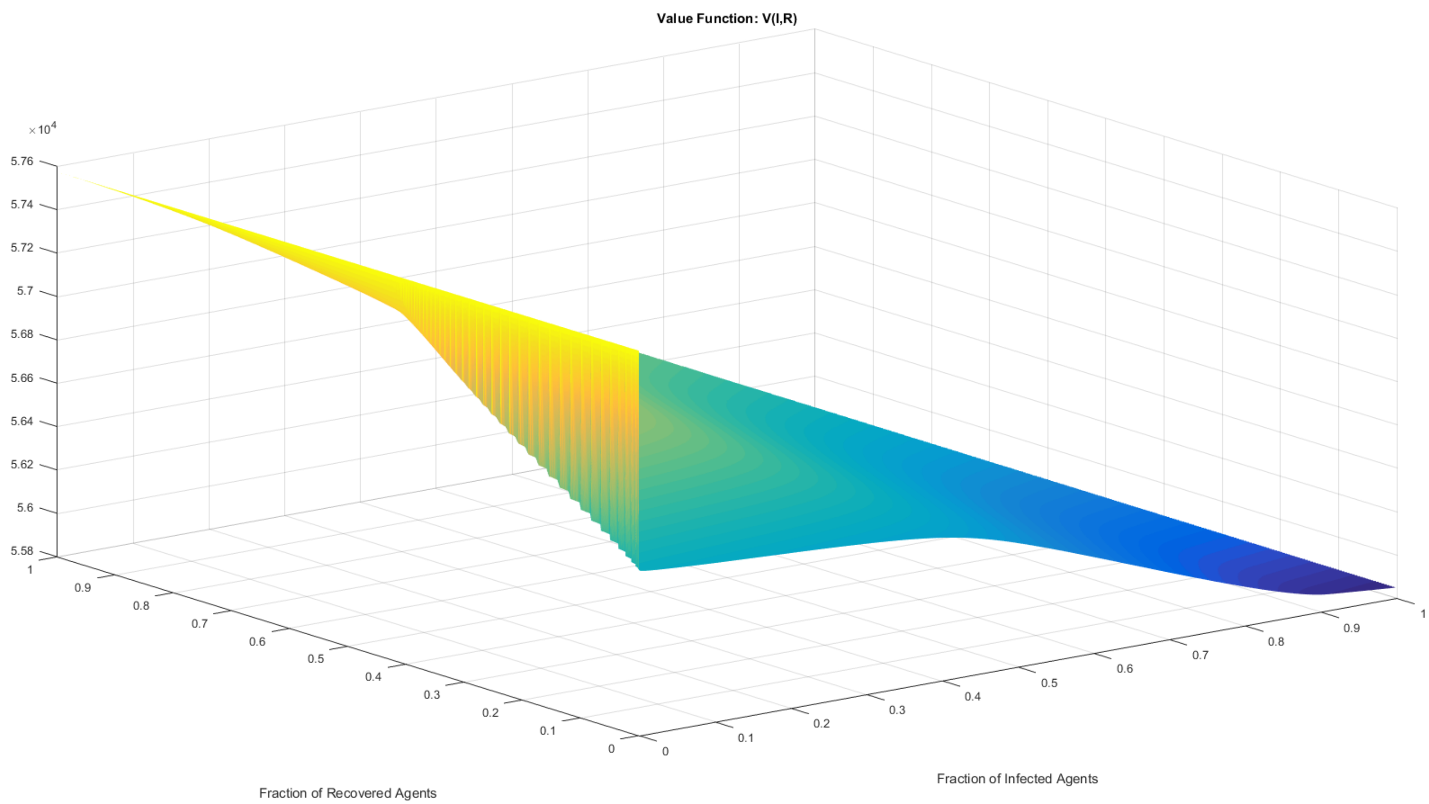


Figure 3: Social planner´s value function $V(I, R)$.

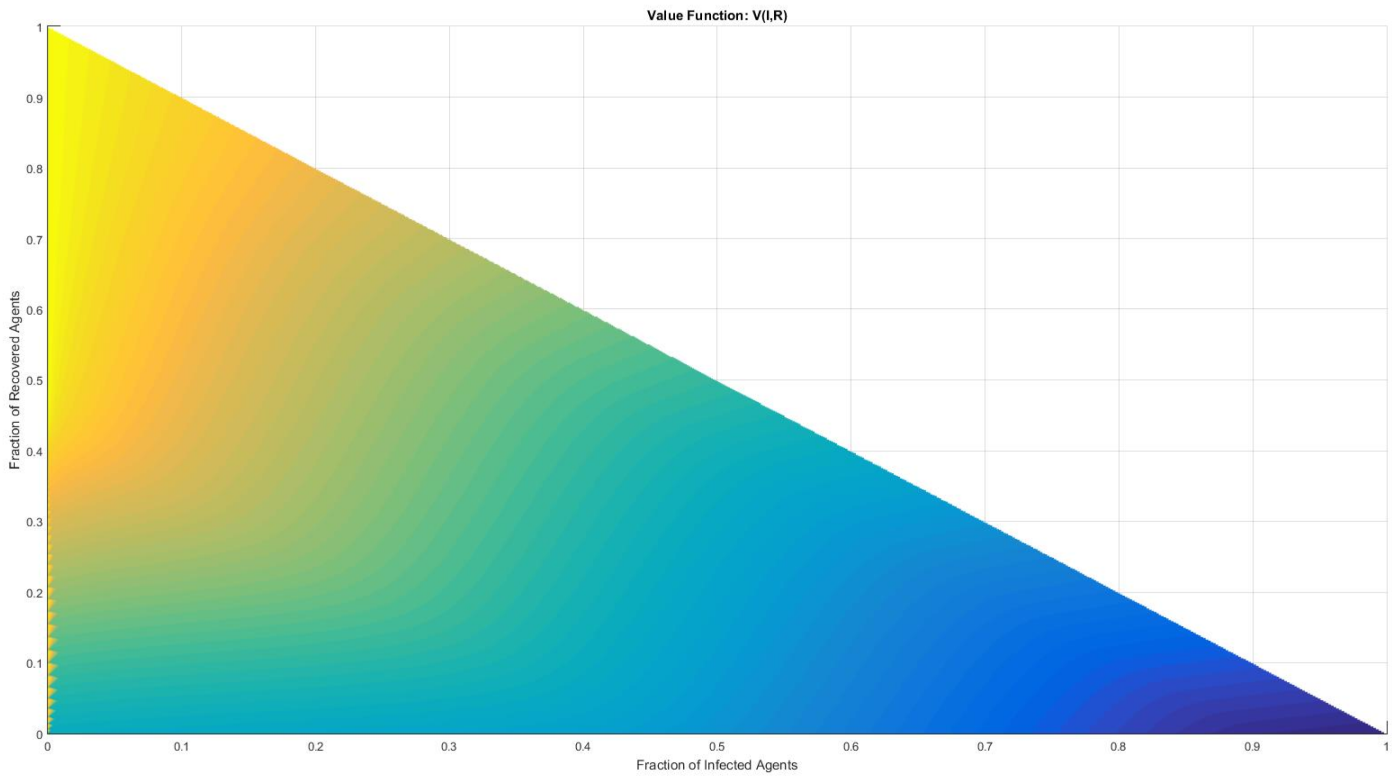


Figure 4: Social planner´s value function $V(I,R)$ heat map.

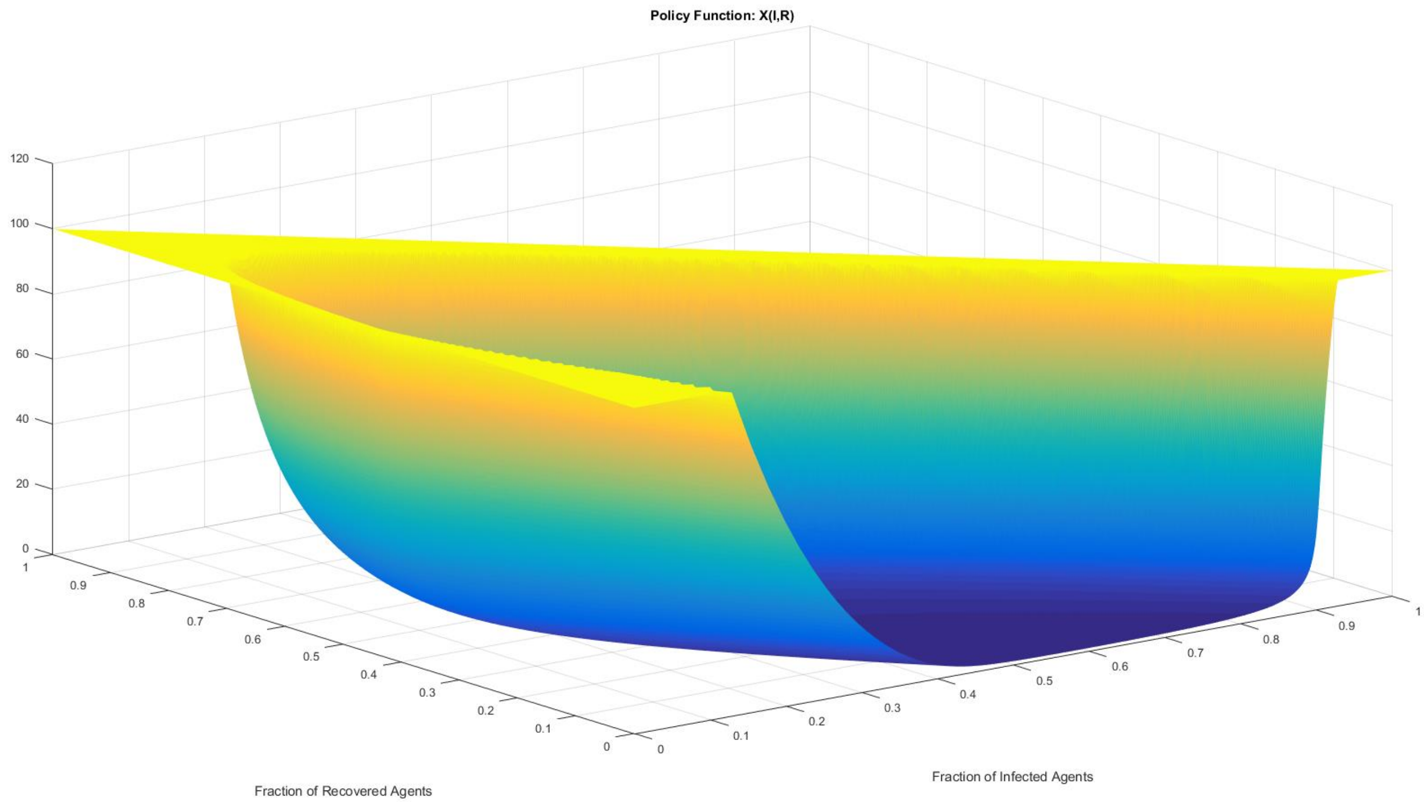


Figure 5: Social planner´s policy function $X(I,R)$.

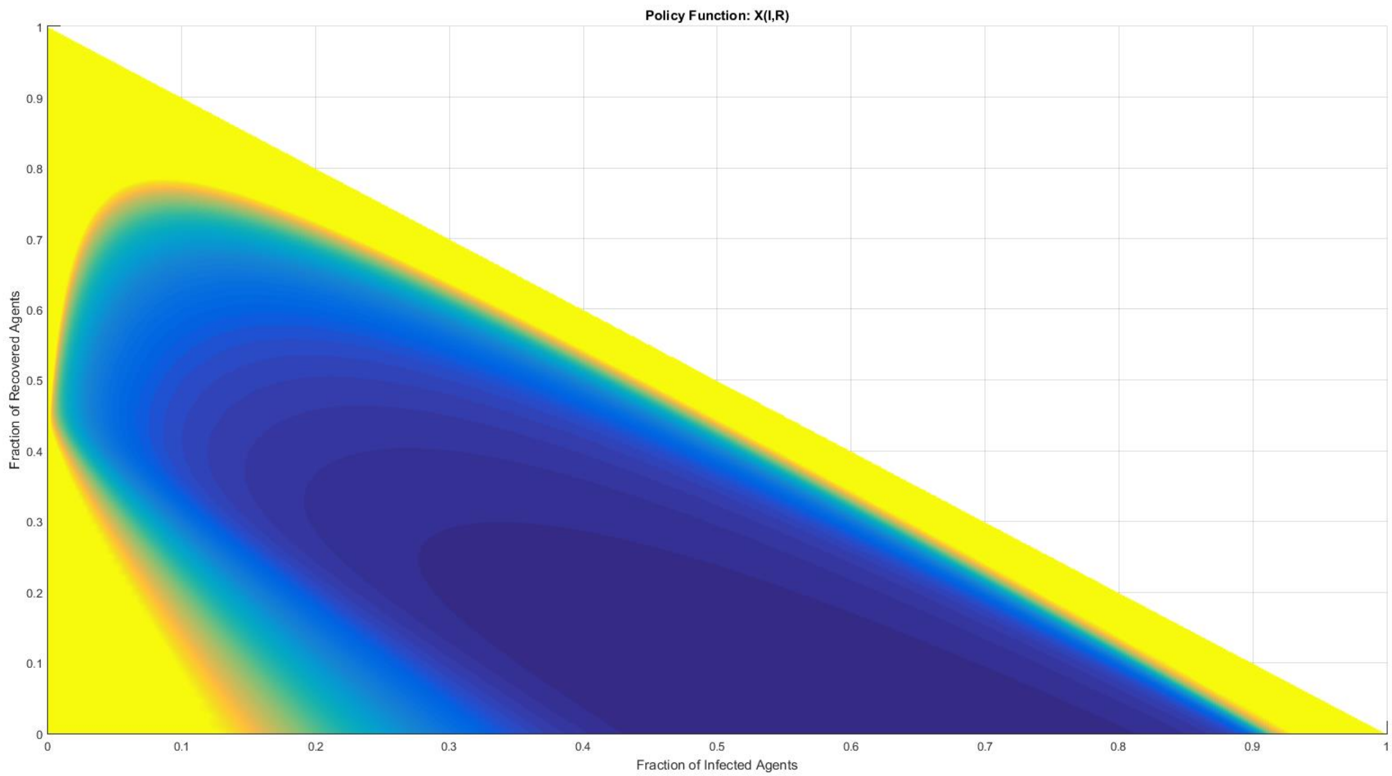


Figure 6: Social planner´s policy function $X(I,R)$ heat map.

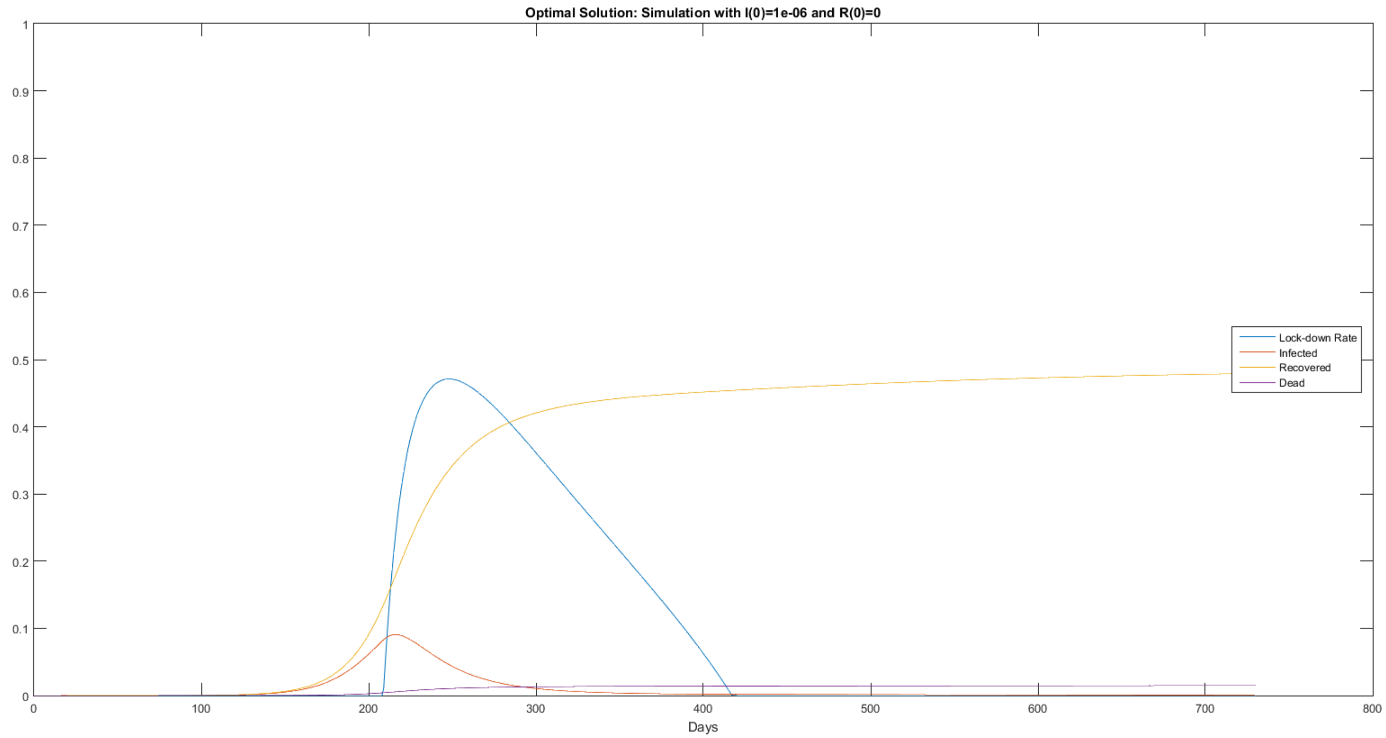


Figure 7: Social planner´s simulation starting with $I(0) = 10^{-6}$ and $R(0) = 0$.

4) Competitive Equilibrium.

Now we set the problem for an individual and solve the symmetric competitive equilibrium. Agents do not know what type they are. The agent takes as given the evolution of the aggregate state variables, I and R, and the aggregate number of contacts $X(I,R)$. Let p be the probability that the agent assigns to being infected conditional on being alive and let q be the probability that the agent assigns to being recovered conditional on being alive. The agent´s problem is

$$(\gamma\mu p+\rho)V(p,q,I,R) = \max_{0<x\leq X_{MAX}}\left\{F(x)+V_p(p,q,I,R)g(x,p,q,I)+V_q(p,q,I,R)h(p,q) + V_I(p,q,I,R)G(X(I,R),I,R)+V_R(p,q,I,R)H(I,R)\right\}$$

$$g(x,p,q,I)=cxI(1-p-q)-\gamma p+\gamma\mu p^2$$

$$h(p,q)=\gamma(1-\mu)p+\gamma\mu pq$$

$$G(X(I,R),I,R)=cX(I,R)I(1-I-R)-\gamma I+\gamma\mu I^2$$

$$H(I,R)=\gamma(1-\mu)I+\gamma\mu IR$$

The first order condition is

$$F_x+V_pg_x\geq 0$$

and the envelope Conditions are

$$\gamma\mu V+(\gamma\mu p+\rho)V_p=V_{pp}g+V_pg_p+V_{qp}h+V_qh_p+V_{Ip}G+V_{Rp}H$$

$$(\gamma\mu p+\rho)V_q=V_{pq}g+V_pg_q+V_{qq}h+V_qh_q+V_{Iq}G+V_{Rq}H$$

$$(\gamma\mu p+\rho)V_I=V_{pI}g+V_pg_I+V_{qI}h+V_{II}G+V_I\left(G_X\frac{\partial X}{\partial I}+G_I\right)+V_{RI}H+V_RH_I$$

$$(\gamma\mu p+\rho)V_R=V_{pR}g+V_{qR}h+V_{IR}G+V_I\left(G_X\frac{\partial X}{\partial R}+G_R\right)+V_{RR}H+V_RH_R$$

Let $x=x(p,q,I,R)$ be the policy function that solves to the previous problem. A symmetric equilibrium is one in which

$$x(I,R,I,R)=X(I,R)$$

So

$$\frac{\partial X}{\partial I}=x_p+x_I,\qquad \frac{\partial X}{\partial R}=x_q+x_R$$

with

$$x_p=\begin{cases}-\dfrac{V_{pp}g_x+V_pg_{xp}}{F_{xx}} & if\ F_x+V_pg_x=0\\ 0 & o/w\end{cases},\qquad x_q=\begin{cases}-\dfrac{V_{pq}g_x+V_pg_{xq}}{F_{xx}} & if\ F_X+V_pg_x=0\\ 0 & o/w\end{cases}$$

$$x_I = \begin{cases} -\dfrac{V_{pI}g_x + V_p g_{xI}}{F_{xx}} & if\ F_X + V_p g_x = 0 \\ 0 & o/w \end{cases}, \qquad x_R = \begin{cases} -\dfrac{V_{pR}g_x}{F_{xx}} & if\ F_X + V_p g_x = 0 \\ 0 & o/w \end{cases}$$

Since we need V_{pp}, V_{pq}, V_{pI} and V_{pR} we have to differentiate the envelope conditions for p and q with respect to p, q, I and R. Let $\upsilon \equiv V_q, \varphi \equiv V_p, \Upsilon \equiv V_R, \Phi \equiv V_I, \upsilon_q \equiv V_{qq}, \upsilon_R \equiv V_{qR}, \varphi_q \equiv V_{pq}, \varphi_p \equiv V_{pp}, \upsilon_I \equiv V_{qI}, \varphi_R \equiv V_{pR}$ and $\varphi_I \equiv V_{pI}$ then we can get the following differential equations

$$\dot{\upsilon} = \left(\gamma\mu p + \rho - h_q\right)\upsilon - g_q\varphi$$

$$\dot{\varphi} = \gamma\mu\frac{F + g\varphi + h\upsilon + G\Phi + H\Upsilon}{\gamma\mu p + \rho} + \left(\gamma\mu p + \rho - g_p\right)\varphi - h_p\upsilon$$

$$\dot{\Upsilon} = (\gamma\mu p + \rho - H_R)\Upsilon - \left(G_X\frac{\partial X}{\partial R} + G_R\right)\Phi$$

$$\dot{\Phi} = \left(\gamma\mu p + \rho - G_X\frac{\partial X}{\partial I} - G_I\right)\Phi - g_I\varphi - H_I\Upsilon$$

$$\dot{\upsilon_q} = \left(\gamma\mu p + \rho - 2h_q\right)\upsilon_q - \left(g_x\frac{\partial x}{\partial q} + 2g_q\right)\varphi_q - g_{qx}\frac{\partial x}{\partial q}\varphi$$

$$\dot{\upsilon_R} = \left(\gamma\mu p + \rho - h_q - H_R\right)\upsilon_R - g_x\frac{\partial x}{\partial R}\varphi_q - g_{qx}\frac{\partial x}{\partial R}\varphi - g_q\varphi_R - \left(G_X\frac{\partial X}{\partial R} + G_R\right)\upsilon_I$$

$$\dot{\varphi_q} = \left(\gamma\mu p + \rho - g_p - h_q\right)\varphi_q - \left(g_x\frac{\partial x}{\partial q} + g_q\right)\varphi_p - g_{px}\frac{\partial x}{\partial q}\varphi - h_p\upsilon_q$$

$$\dot{\varphi_p} = \left(\gamma\mu p + \rho - 2g_p - g_x\frac{\partial x}{\partial p}\right)\varphi_p - g_{px}\frac{\partial x}{\partial p}\varphi - 2h_p\varphi_q$$

$$\dot{\upsilon_I} = \left(\gamma\mu p + \rho - h_q - G_X\frac{\partial X}{\partial I} - G_I\right)\upsilon_I - \left(g_x\frac{\partial x}{\partial I} + g_I\right)\varphi_q - g_q\varphi_I - \left(g_{qI} + g_{qx}\frac{\partial x}{\partial I}\right)\varphi - H_I\upsilon_R$$

$$\dot{\varphi_R} = \left(\gamma\mu p + \rho - g_p - H_R\right)\varphi_R - g_x\frac{\partial x}{\partial R}\varphi_p - g_{px}\frac{\partial x}{\partial R}\varphi - h_p\upsilon_R - \left(G_X\frac{\partial X}{\partial R} + G_R\right)\varphi_I + \gamma\mu\Upsilon$$

$$\dot{\varphi_I} = \left(\gamma\mu p + \rho - g_p - G_X\frac{\partial X}{\partial I} - G_I\right)\varphi_I - \left(g_x\frac{\partial x}{\partial I} + g_I\right)\varphi_p - \left(g_{px}\frac{\partial x}{\partial I} + g_{pI}\right)\varphi - H_I\varphi_R - h_p\upsilon_I + \gamma\mu\Phi$$

Again, we have a continuum of steady states, but only the ones with $R_{SS} > R_{MIN} = 1 - \frac{\gamma}{cX_{MAX}}$ are stable. The stable steady states in a symmetric equilibrium are

$$\upsilon_{SS} = 0$$

$$\varphi_{SS} = -\gamma\mu \frac{F_{MAX}}{\rho(\rho - g_p)}$$

$$\Upsilon_{SS} = 0$$

$$\Phi_{SS} = \frac{g_I}{\rho - G_I}\varphi_{SS} = \frac{cX_{MAX}(1 - R_{SS})}{\rho - cX_{MAX}(1 - R_{SS}) + \gamma}\varphi_{SS}$$

$$\upsilon_{q,SS} = 0$$

$$\upsilon_{R,SS} = 0$$

$$\varphi_{q,SS} = 0$$

$$\varphi_{p,SS} = 0$$

$$\upsilon_{I,SS} = \frac{g_{qI}}{\rho - G_I}\varphi_{SS}$$

$$\varphi_{R,SS} = 0$$

$$\varphi_{I,SS} = \frac{g_{pI}\varphi_{SS} + h_p\upsilon_{I,SS} - \gamma\mu\Phi_{SS}}{\rho - g_p - G_I}$$

Let

$$Y = [R \quad I \quad \upsilon \quad \varphi \quad \Upsilon \quad \Phi \quad \upsilon_q \quad \upsilon_R \quad \varphi_q \quad \varphi_p \quad \upsilon_I \quad \varphi_R \quad \varphi_I]'$$

and Y_{SS} be the corresponding vector with the steady state values. We then take the linear approximation around the steady state

$$\dot{Y} = J(Y - Y_{SS})$$

The eigenvalues of the matrix J are $\{G_I, 0, \rho - G_I, \rho - G_I, \rho - G_I, \rho, \rho, \rho, \rho, \rho, \rho, \rho, \rho\}$. Again we have that if $R_{SS} > R_{MIN} = 1 - \frac{\gamma}{cX_{MAX}}$ then $G_I < 0$ and the system has only one stable root and is saddle path stable. The 13 by 13matrix, J, is shown in Table 1.

We now solve numerically the problem of the decentralized economy with the same parametrization as in the previous section. Figures 8 through 12 present the results. Figure 13 presents a simulation for the decentralized economy starting at $I(0) = 1e - 6$ $R(0) = 0$.

0	H_I	0	0	0	0	0	0	0	0	0	0	0
0	G_I	0	0	0	0	0	0	0	0	0	0	0
0	$-g_{qI}\varphi_{SS}$	ρ	0	0	0	0	0	0	0	0	0	0
0	$\frac{\gamma\mu}{\rho}G_I(\varphi_{SS}+\Phi_{SS})-\left(\frac{\gamma\mu}{\rho}\right)^2 F - (\gamma\mu+g_{pI})\varphi_{SS}$	$-h_p$	ρ	0	0	0	0	0	0	0	0	0
0	$-G_{RI}\Phi_{SS}$	0	0	ρ	0	0	0	0	0	0	0	0
$-G_{IR}\Phi_{SS} - g_{Iq}\varphi_{SS}$	$-\gamma\mu\Phi_{SS}-g_{Ip}\varphi_{SS}$	0	$-g_I$	$-H_I$	$\rho - G_I$	0	0	0	0	0	0	0
0	0	0	0	0	0	ρ	0	0	0	0	0	0
0	$-G_{RI}\upsilon_{I,SS}$	0	0	0	0	0	ρ	0	0	0	0	0
0	0	0	0	0	0	$-h_p$	0	ρ	0	0	0	0
0	0	0	0	0	0	0	0	$-2h_p$	ρ	0	0	0
$-G_{IR}\upsilon_{I,SS}$	$-G_{II}\upsilon_{I,SS}-g_{qI}\varphi_{I,SS}$	0	$-g_{qI}$	0	0	0	$-H_I$	$-g_I$	0	$\rho - G_I$	0	0
$-G_{RI}\varphi_{I,SS}$	0	0	0	$\gamma\mu$	0	0	$-h_p$	0	0	0	ρ	0
$-h_{pq}\upsilon_{I,SS}$	$(\gamma\mu - g_{pp} - g_{pI} - G_{II})\varphi_{I,SS}$	0	$-g_{pI}$	0	$\gamma\mu$	0	0	0	$-g_I$	$-h_p$	$-H_I$	$\rho - G_I$

Table 1: J Matrix.

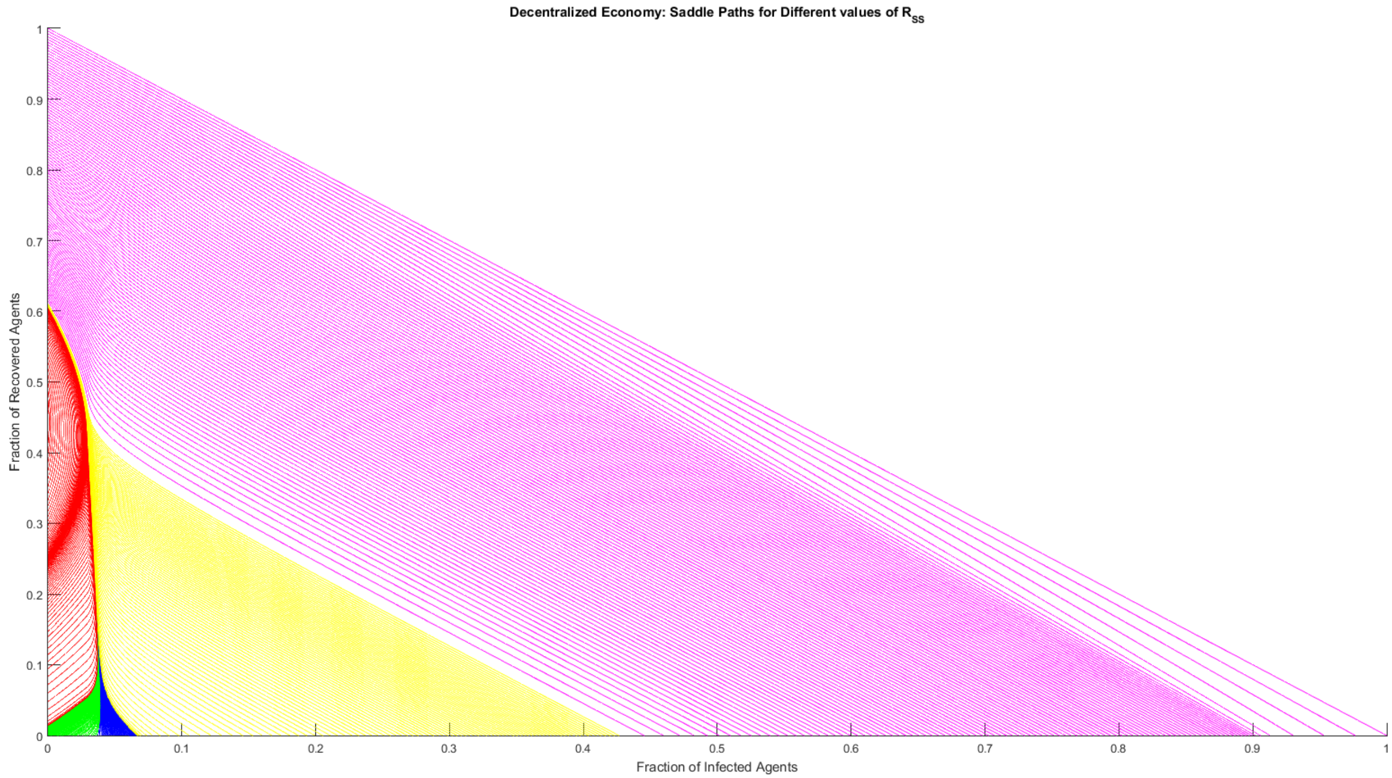


Figure 8: Saddle paths for the decentralized economy in a symmetric equilibrium for different steady states.

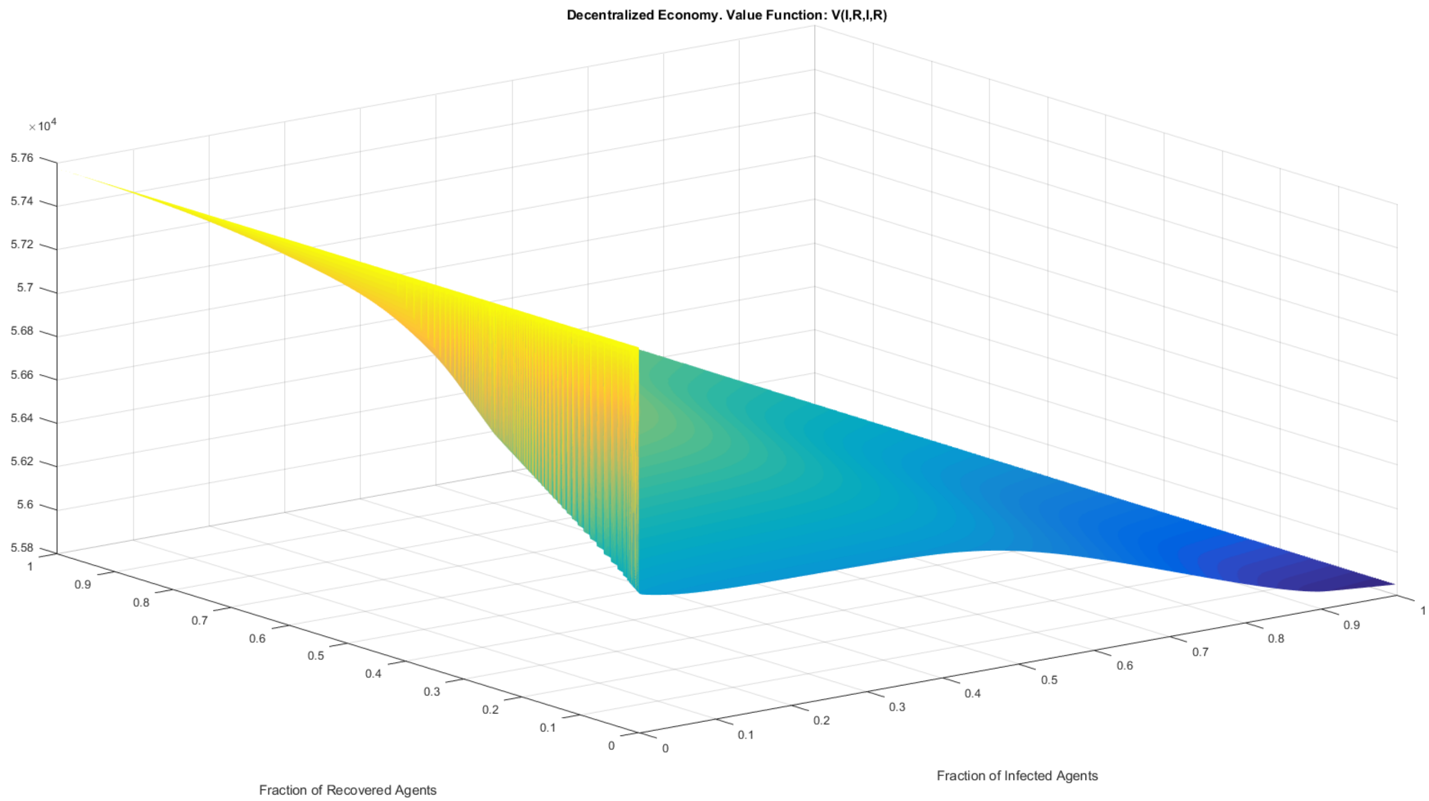


Figure 9: Value function for the decentralized economy in a symmetric equilibrium, $V(p = I, q = R, I, R)$.

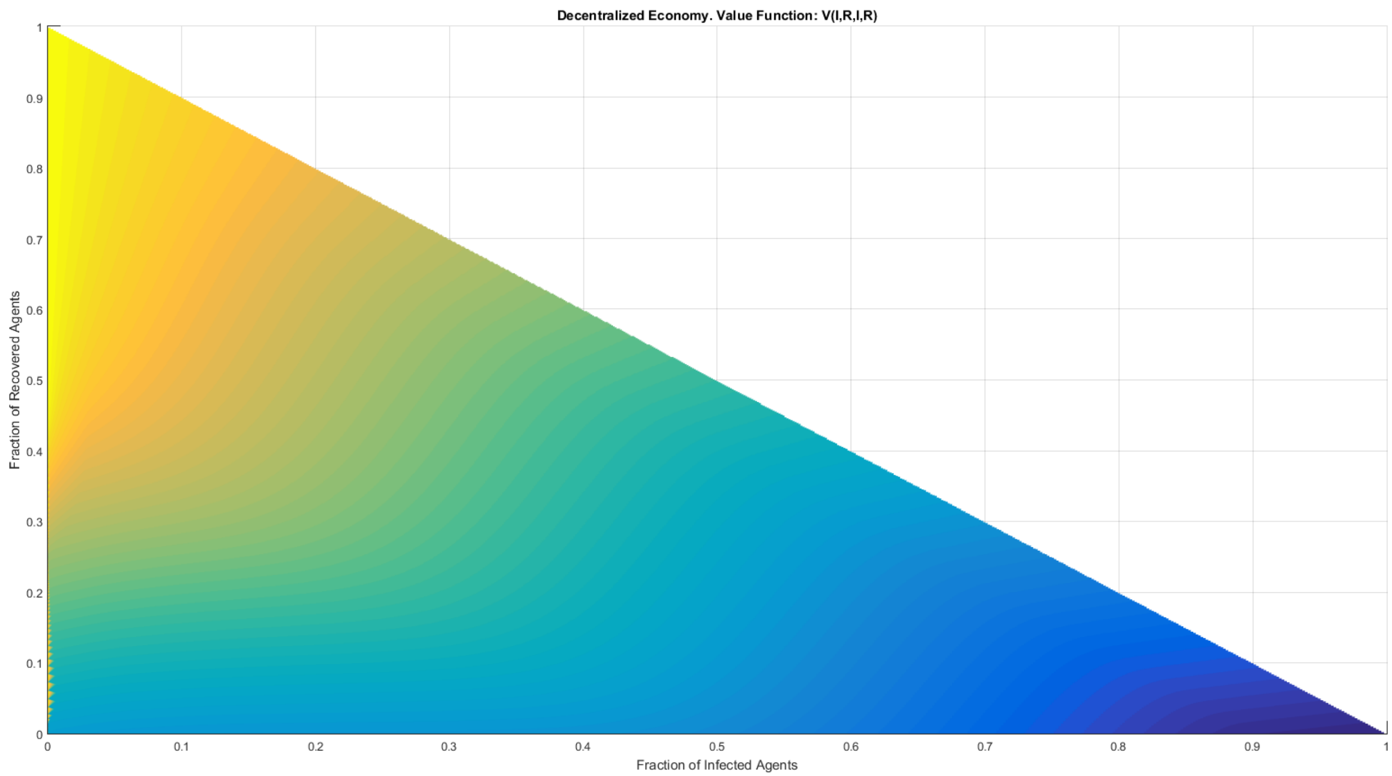


Figure 10: Heat map for the value function for the decentralized economy in a symmetric equilibrium, $V(p = I, q = R, I, R)$.

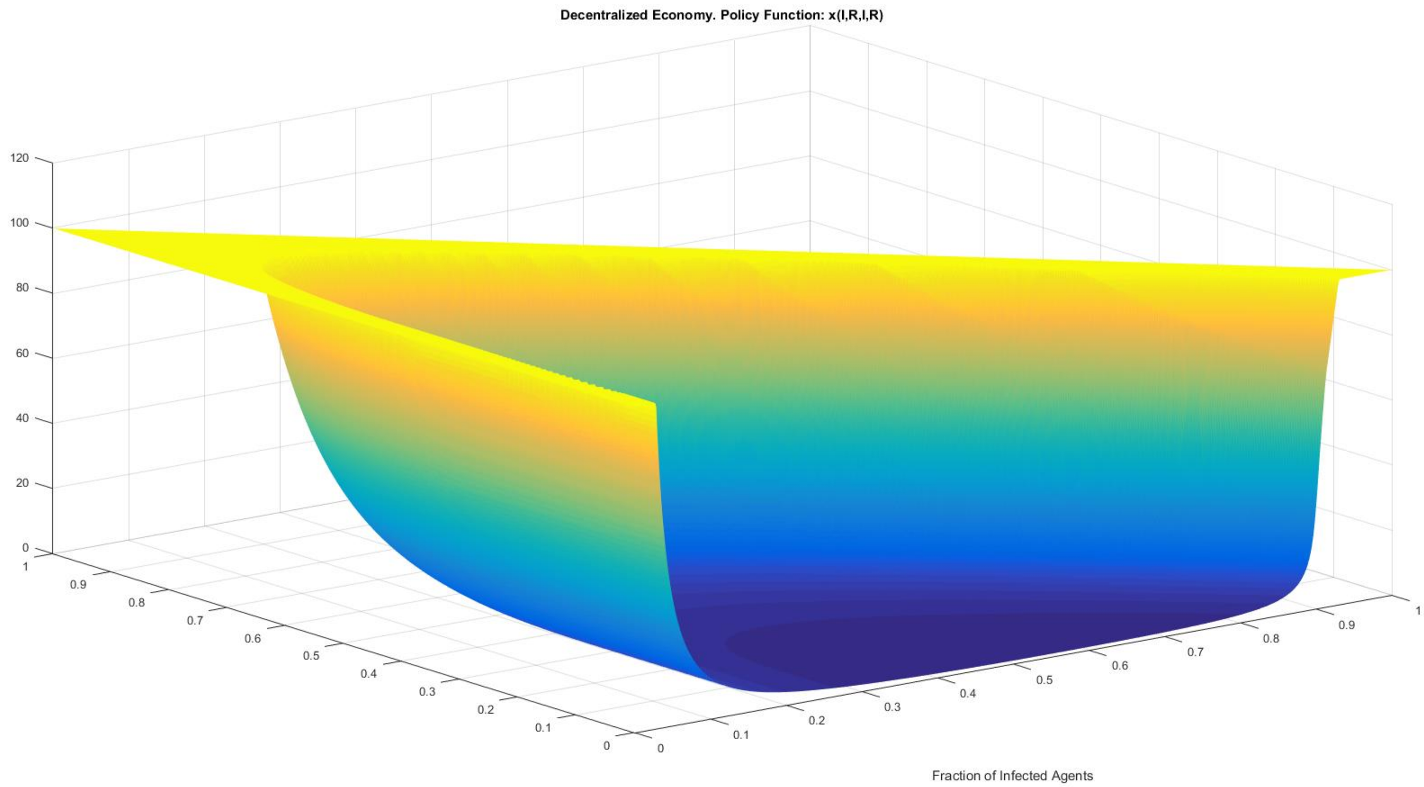


Figure 11: Policy function for the decentralized economy in a symmetric equilibrium, $x(p = I, q = R, I, R)$.

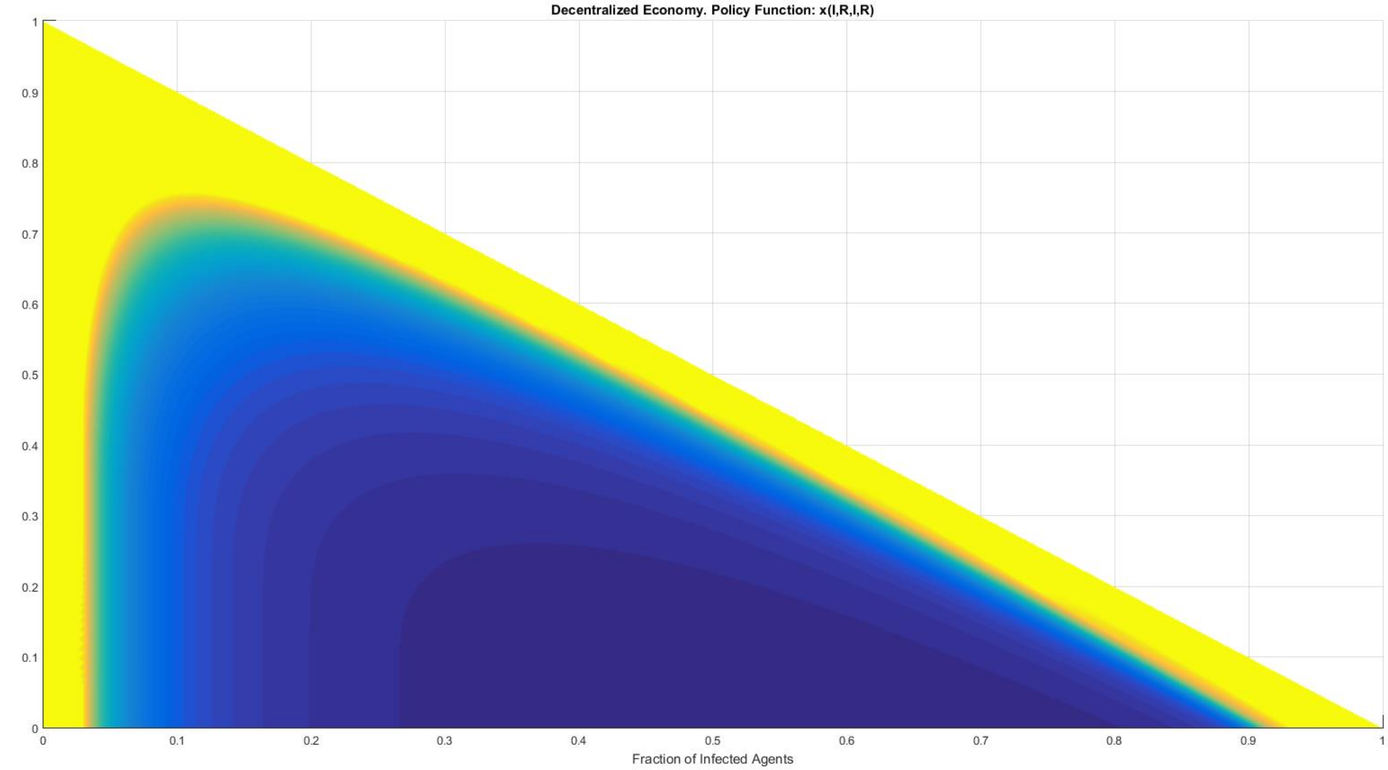


Figure 12: Heat map for the policy function for the decentralized economy in a symmetric equilibrium, $V(p = I, q = R, I, R)$.

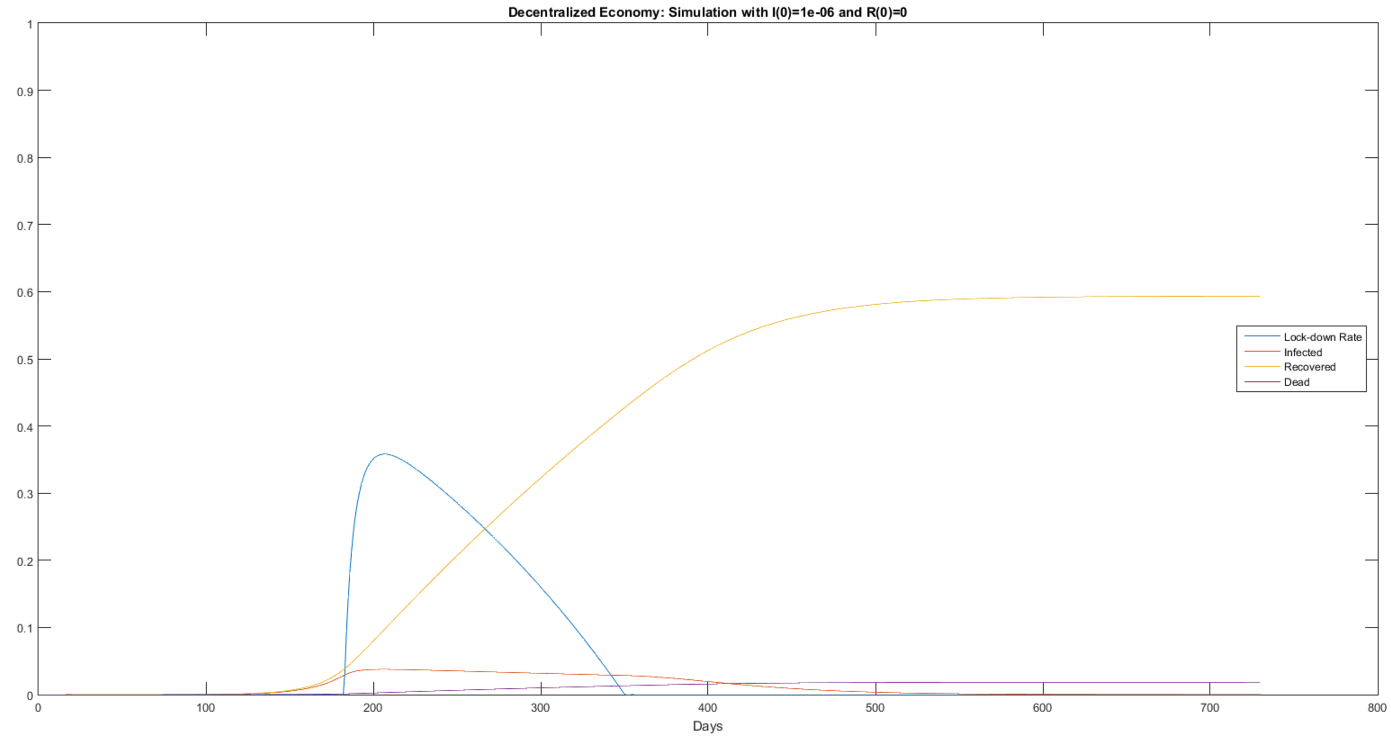


Figure 13: Simulation for a decentralized economy starting at $I(0) = 10^{-6}$ and $R(0) = 0$.

5) Conclusions

Epidemics have significant economic costs both in terms of lives lost and in forgone output while trying to control the epidemic. This is true in a decentralized economy and in a centralized one. Under our parametrization the number of deaths from an epidemic is 1.5% of the original population in the centralized economy and around 1.8% in a decentralized economy. In an infinitely lived agent model (absent the epidemic) this represents a permanent loss of output. Optimal or decentralized lockdowns also add to the cost of and epidemic. In our simulations we find that lockdowns can last around 200 days and affect between 40 to 50% of the economy. These results are parameter specific. Numerical results show that a lower curvature of the production function yields a shorter but more severe lockdown as there are less incentives to smooth things out.

We think that policy recommendations regarding an epidemic should be conducted with care since there is great uncertainty, but we should always keep in mind individual incentives when doing so. In our model we find that the optimal lockdown policy is not about forcing people to stay at home but to force people to go to work early on when the epidemic starts to spread.

Our model is stripped from many of the various factors that might be relevant when faced with an epidemic, like capacity constraints, vaccines and other aspects that have been discussed in the literature, but we think that our framework is flexible enough (although computationally intensive) to incorporate them.

6) References.

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